“Income Inequality and Economic Growth”

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Abstract

This thesis gives a summary of the empirical and theoretical literature on the relation between income inequality and economic growth. The focus concentrates on two approaches: the Political Economy approach and the Capital Market Imperfection approach. The diverging results of these approaches are then combined to a single theory that tries to explain this complex relationship between growth and inequality. This should lead to a better understanding of the mechanisms that influence both, income inequality and economic growth and can help to draw useful policy conclusions.

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Part I. Introduction

The question how income inequality affects economic growth has always been an important one - not only for society but also for politics. Income inequality is usually not meant to be a good thing. Economists as well as social scientists have been debating on how income inequality effects economic growth for more than two centuries.

Already the classical economists stated that income inequality fosters growth in the post-industrialized state. Later, the neoclassical approach suggested that inequality does not effect economic development but income inequality is affected by the growth process. The modern perspective now states that income inequality is an important force that influences the growth process.

The modern perspective boosts discussions on how to react on rising inequality. A nice example was made by the IMF recently (see Berg and Ostry(2011)). Imagine that there are 1000 boats that represents the US and the length of each boat represent income: While in the 1970s the average boat was 3.7 meters and the longest 76.2 meters, in 2000 the average boat was 4.6 meters while the largest boat was 335.3 meter. As the authors mentioned: "When a handful of yachts become ocean liners while the rest remain lowly canoes, something is seriously amiss". (Berg and Ostry(2011), IMF global economic forum) The rich got much richer while the average worker had a modest income growth. Such immense patterns of inequality did not occur since the late 1920s and it obviously raises the question of redistribution. On the other hand it also raises the question whether there is a link between economic crises and income inequality. The Great Depression of the early 1930s and the Great Recession of the late 2000s might have been affected by the high level of inequality. Since inequality affects the economic process it will for sure influence such a crisis.

Whether redistribution from the rich to the poor has a positive or a negative effect on economic growth is therefore an important question for policy makers nowadays. Empirical studies differ widely in their results. There is no clear empirical evidence that suggests that a simple relation between income inequality and economic growth exists. The same is true for economic theories. While some suggest a positive relation, most of them show a negative one.

This thesis gives an overview on the relation of income inequality and economic growth. It is arranged as follows: The main empirical findings are summarized in the second part. The third part combines the main theories on income inequality and economic growth and introduces some theoretical models. The fourth part points out the policy implications of the theory and the fifth part finally gives the major conclusions.

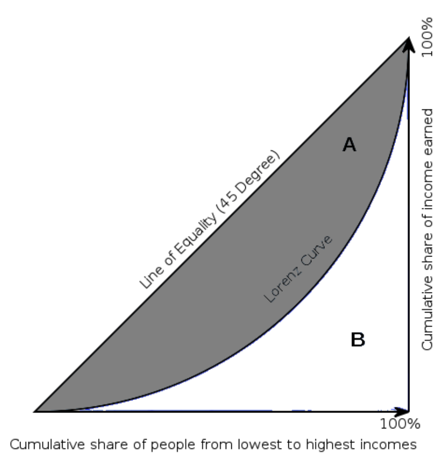
Part II. Empirical Studies

The empirical studies on the relationship between income inequality and economic growth are numerous. The results of this research are far away from being unique. While older studies suggests a negative relation especially newer research shows a positive relation between income inequality and economic growth.

The question of how to measure inequality is most important for empirical analysis. There are many different methods of measuring inequality and there is no clear recommendation which measure to use. Most frequently the Gini coefficient and the share of income are used but there are many other measures that can be used. The following paragraph will summarize and slightly explain these methods (for further informations see: Cowell(1995)):

- The **Gini coefficient** is based on the Lorenz curve which plots on the y axes the cumulative income that is earned by the bottom x% of the population. The 45°-line will therefore be total equality in society. The Gini coefficient is the area between the 45°-line and the Lorenz curve (A) compared to the area under total equality (A+B). Obviously a Gini coefficient close to 0 will imply a equal income distribution while a Gini coefficient close to 1 will imply unequal distribution of income.

Fig. 1: The Gini Coefficient



- The **Hoover index** states which fraction of the income has to be redistributed to get total equality in society. Obviously a Hoover coefficient close to zero indicates an equal society while an Hoover coefficient close to 1 indicates high inequality in the income distribution.

- The **Theil index** is a measure of entropy in a system and can not only be used for inequality. It is derived by using the highest possible entropy and subtract the entropy of the data. Maximum entropy occurs when people can not be distinguished by their income, therefore a Theil coefficient of 0 indicates perfect equality while 1 indicates perfect inequality.
- The **Atkinson index** is based on the theoretical framework of the Theil index but has an interesting feature. One can choose the level of inequality aversion in order to make the coefficient more sensitive to changes in different parts of the income distribution.
- The **ratio of percentiles** is also often used to measure inequality. One can use for example the ratio between the median and the 10th percentile to measure losses or gains of the lower income earners.
- The **share of income** is based on dividing the population in subpopulations and see for what percentage of total income it accounts for. The income share is usually used to show changes in the bottom-end and top-end income distribution while e.g. the Gini coefficient reflects the overall effect.

In recent papers (e. g. Voitchovsky(2005) or Foellmi and Oechslin(2008)) the way of measuring income inequality by the Gini coefficient is criticized.

A splitting of the empirical part into 4 sections seems to be reasonable. Section 1 presents the early literature and especially the findings of Kuznets(1955,1963). Section 2 shortly summarizes the empirical research based on cross-country data, while section 3 will introduce the main results of panel-data based studies. Section 4 will then be a short summary and will also give a critical view on the empirical research made so far.

1 Kuznets Model

Kuznets(1955) is one of the first papers that deals with the question of whether inequality in the distribution of income increases or decreases economic growth. It is not only an empirical study, Kuznets also came up with a simple model that explains his empirical findings.

He used data from the US, UK and Germany (developed countries) and India, Ceylon and Puerto Rico (developing countries). By comparing the deciles of developed and underdeveloped countries, Kuznets(1955) found two statistical patterns of the income distribution:

1. Usually underdeveloped countries have no middle class, meaning that most of the population lies far below the average income level while a small group receives relative high incomes. Developed countries typically show a large group that receives income levels beyond the average and a top group that possesses a low income share (compared to the top group in the underdeveloped countries). This implies that inequality is higher in underdeveloped than in more developed countries. Table 1 shows that in India, Ceylon and Puerto Rico income was distributed more unequally compared to the US and the UK after World War II.

2. Annual income before taxes and without government transfers moves from an unequal to a more equal distribution after the 1920s in developed countries. Table 1 shows that phenomenon for the US and the UK.

Tab. 1: EMPIRICAL FINDINGS of Kuznets(1955) for developed and developing countries

United States	1.+2. quintile	top quintile	top 5%
1929	13.5%	55%	31%
ϕafter WWII	18%	44%	20%

United Kingdom	lower 85%	top 5%
1880	41%	46%
1913	43%	43%
1929	46%	33%
1938	-	31%
1947	55%	24%

post-WWII data	lower 3 quintiles	top quintile
India	28%	55%
Ceylon	30%	50%
Puerto Rico	24%	56%
United States	34%	44%
United Kingdom	36%	45%

The numbers in the tables show the income shares of the given group in the given country.

Recognizing that there is more inequality in the distribution of income of developing countries led Kuznets(1955) to the following conclusions:

- Higher inequality in the income distribution is associated with lower average income per capita. Saving in underdeveloped countries is more scarce than in developed. Only a really small group will be able to save. This might explain the missing middle class in the income structure in underdeveloped countries.

- In former times, underdeveloped countries started at the same levels of GDP per capita as developed countries did. Therefore the unequal income structure in developing countries goes along with a low rate of growth in income per capita.

- The inequality in the distribution of income in underdeveloped countries did not scale down during the 1930's and 1940's. This is due to the effect of accumulation of savings at the peak of the income distribution and the weak government-support for low income groups in these countries.

According to Kuznets(1955) there are at least two forces that lead to increasing inequality in the long-run:

- The concentration of savings in the higher income groups leads to a concentration of income-producing assets in the hand of the top quintile group. Therefore inequality should increase.

- The economic development influences the income distribution. The population can be divided in a part that works in the industrial sector and a part that works in the agricultural sector. In the growth process, a country usually moves from an agricultural society to an industrial one. The average income of the population in the agricultural sector is usually lower than in the industrial sector. Additionally, inequality is usually lower in the agricultural sector. During the growth process, population shifts from agriculture to industry, meaning that the more unequal distribution (in the industrial sector) gains weight. Additionally the relative difference in income between the agricultural and the industrial sector increases due to a faster increase in productivity in the industrial sector. This overall means, inequality in the income distribution should increase.

Thus the question arises, why empirically observed inequality declines and especially why the income share of the lower income-group increases in developed countries ? What are the factors that counteract to the accumulation of savings?

- First, when most of the population works in the industrial sector, the inequality will decline because most of the people work now in the industry sector and the income gap between the sectors loses importance in determining the inequality.

- Second, as democracy develops in countries, the general view on rising inequality might change at some level of inequality. Policy makers might intervene to curb the capital concentration in the hand of a small group by introducing taxes and capital levies.

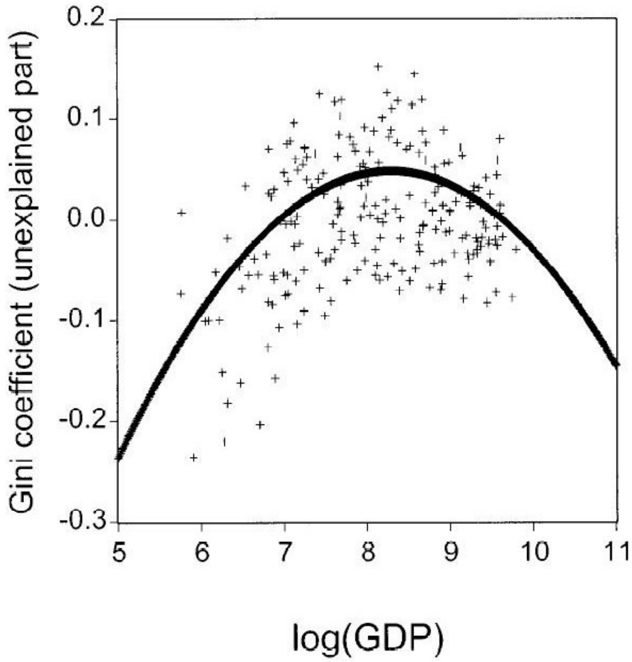
- Third, in a more industrialized economy, the younger industry grows faster than the old one, leading to a relative decrease in wealth of old entrepreneurs. In this context, Kuznets(1955) states: "The successful great entrepreneurs of today are rarely the sons of the great and successful entrepreneurs of yesterday."[23]

- Fourth, taking the high services income of a small group into account, it can often be traced back to individual excellence. For the next generation, it will be hard to keep the level of service income. Additionally, the main reason for income increase is the inter-industrial shift from low-income to high-income industry. This is more limited for the high income group than for the low income group.

These arguments do not indicate an overall effect on inequality - inequality might be still increasing but it could also be decreasing. Kuznets(1955) concludes: "One can say, that the basic factor militating against the rise in upper-income shares that would be produced by the cumulative effects of concentration of savings, is the dynamism of a growing and free economic society" [23]

Inequality starts to widen at the beginning of the industrialization till the industrialization stabilizes and a reduction of inequality occurs. This phenomenon is known as the KUZNETS CURVE. Not only because it was more a guesswork than an empirical study, the validity is still not clear. Deininger and Squire(1998) showed that especially in underdeveloped countries the Kuznets relation can often not be found, while Barro(2000) stated the Kuznets curve as an empirical regularity. Figure 2 shows the estimated Kuznets curve of Barro(2000).

Fig. 2: Kuznets Curve of Barro(2000)



Source: Barro, R. (2000). Inequality and Growth in a Panel of Countries. Journal of Economic Growth , pp. 5-32.

The question, whether the development of European countries and the US can be repeated in underdeveloped countries, was not answered by Kuznets. It should be noted that the social and political background in underdeveloped countries is and was entirely different from that in the European countries and the US during the process of industrialization.

2 Cross-Country Studies

After Kuznets's findings most studies focused on cross-section analysis due to lack of data for panel analysis. Especially in the 1990's a lot of research was carried out. Most of it came to the same conclusion: Inequality harms growth.

Persson and Tabellini(1994) performed a panel regression on historical data from 1830 to 1985 and then a cross-country regression on post-war data from 1960 to 1985.

For the historical data that includes nine countries (8 European and the US), they split the sample in periods of 20 years to get panel data. A simple growth model was estimated:

$$GROWTH_{i,t} = \beta_0 + \beta_1 * INCSH_{i,t} + \beta_2 * NOFRAN_{i,t} + \beta_3 * SCHOOL_{i,t} + \beta_4 * GDPGAP_{i,t} + \epsilon_{i,t} \quad (1)$$

$GROWTH$ is the average growth rate of GDP per capita. $INCSH$ is the share in personal income of the top quintile of the population and is used as a proxy for inequality. The higher $INCSH$, the greater the inequality of the population. $SCHOOL$ is a weighted average of the shares of relevant age groups that are enrolled in different types of schooling. The higher the level of schooling, the higher the given weight. $GDPGAP$ is the ratio of GDP per capita of the country and the highest GDP per capita in the sample and is a proxy for the level of development of a country. $NOFRAN$ is the ratio of enfranchised age and sex groups and is based on the theoretical model they introduced. The index i is a country index and t a period index.

Tab. 2: GROWTH REGRESSION (historical data) of Persson and Tabellini (1994)

TABLE 2—REGRESSIONS FOR GROWTH

Independent variable	Regression (i)	(ii)	(iii)	(iv)
Constant	5.263	7.206	6.256	6.465
	(2.659)	(5.723)	(4.066)	(6.899)
INCSH	−3.481	−6.911	−6.107	−6.409
	(−1.017)	(−3.074)	(−2.234)	(−3.963)
NOFRAN	−0.782		−0.011	
	(−0.670)		(−0.018)	
SCHOOL	2.931		0.316	
	(0.913)		(0.204)	
GDPGAP	−2.591	−2.695	−1.720	−1.728
	(−2.739)	(−2.696)	(−2.708)	(−2.778)
Number of observations:	38	38	56	56
$\bar{R}^2$:	0.294	0.298	0.269	0.296
SEE:	0.931	0.929	0.882	0.866

Notes: The table reports ordinary least-squares regressions; t values are shown in parentheses. SEE = standard error of the estimate.

Source: Persson, T., & Tabellini, G. (1994). Is Inequality Harmful for Growth? American Economic Review 84 , pp. 600-621.

The striking result of the regression is that the coefficients of inequality ($INCSH$) are always negative and significant in 3 of the 4 cases where different models and samples are used. The coefficient of $GDPGAP$ is always significant and has a negative sign. That indicates a convergence to a certain GDP level over time. All the other coefficients are not significant and do not yield to any interesting conclusion. After sensitivity testing, Persson and Tabellini(1994) conclude that their results are robust. Reverse causation can be ruled out but there is still the possibility of omitting variables.

For the post-war data which includes 56 countries, the regression model is quite similar:

$$GROWTH_i = \beta_0 + \beta_1 * MIDDLE_i + \beta_2 * GDP_i + \beta_3 * PSCHOOL_i + \epsilon_i \quad (2)$$

i is the country index. The income share of the third(middle) quintile, which includes the median, is used as a proxy for inequality and is denoted as $MIDDLE$. That means, the greater equality, the greater the income share of the middle quintile and therefore the expected sign is positive. Typical control variables as initial GDP per capita and a human resource variable $PSCHOOL$ that measures the share of the relevant age group attending primary school are added to the regression.

The results of the post-war data which are also split in democracies and non-democracies are even more striking than the former results and are listed in table 3.

Tab. 3: GROWTH REGRESSION (post-war data) of Persson and Tabellini (1994)

TABLE 5—REGRESSIONS FOR GROWTH

	(i) Whole sample	(ii) Democracies	(iii) Nondemocracies	(iv) Whole sample
Constant	−2.589	−5.159	0.949	0.949
	(2.359)	(−3.363)	(0.526)	(0.572)
MIDDLE	0.189	0.326	−0.072	−0.072
	(2.350)	(3.235)	(−0.559)	(−0.608)
GDP	-5.3×10^{-4}	-5.8×10^{-4}	-1.7×10^{-3}	-1.7×10^{-3}
	(−3.070)	(−3.579)	(−2.967)	(−3.229)
PSCHOOL	0.041	0.049	0.057	0.057
	(4.432)	(3.627)	(3.119)	(3.396)
DEMOCRACY				−6.108
				(−2.624)
MIDDLEDM				0.398
				(2.489)
GDPDM				0.001
				(2.028)
PSCHOOLDM				−0.008
				(−0.377)
Number of observations:	49	29	20	49
$\bar{R}^2$:	0.32	0.46	0.31	0.44
SEE:	1.483	1.265	1.466	1.347

Notes: The table reports ordinary least-squares regressions; t values are shown in parentheses. SEE = standard error of the estimate.

Source: Persson, T., & Tabellini, G. (1994). Is Inequality Harmful for Growth? American Economic Review 84 , pp. 600-621.

All of the coefficients have the expected sign and are in most cases significant. The effects of a one standard deviation increase of equality, that is an increase by 3.1 points in the variable $MIDDLE$, will increase the growth rate by 0.58%. An interesting side result can be seen in column 2 and 3. The effect of inequality and growth is negative and significant in democracies, but the relation is positive for non democracies but not significant. The different results for democracies and non-democracies is perfectly in line with theory. It suggests that there is a negative relation in democracies. But there might be no relation between income inequality and economic growth in non-democracies simply because inequality is connected to economic growth via the median voter theorem This can not be used in non democratic regimes.

Another cross-country study has been done by Alesina and Rodrik(1994). They based their argumentation on the theoretical fact that higher inequality leads to more redistribution. That causes economic distortions and therefore reduces growth. The attempt is to measure the direct effect of inequality on growth.

Their data is split into a low-quality sample that includes 70 countries and a high quality sample that only includes 49 countries. Most of them are OECD countries but also some developing countries are included. To solve the problem of simultaneity, the time horizon is first chosen form 1960 to 1985 and then from 1970 to 1985. Additionally the two-stage least squares method was used.

Typical control variables are included in the growth equation:

$$GROWTH_i = \beta_0 + \beta_1 * GDP60_i + \beta_2 * PRIM60_i + \beta_3 * Gini60_i + \epsilon_i \quad (3)$$

The initial GDP per capita is denoted as $GDP60$ and included for convergence reasons and the primary school enrollment rate $PRIM60$ is used as a proxy for the initial level of human capital.

Tab. 4: Growth regression of Alesina and Rodrik(1994)

	High-quality sample (N = 46)		Largest possible sample (N = 70)		Largest possible sample			
					(N = 49)	(N = 41)		
	OLS (1)	TSLS (2)	OLS (3)	TSLS (4)	OLS (5)	OLS (6)	OLS (7)	OLS (8)
Const.	3.60	8.66	1.76	6.48	3.71	6.22	6.24	6.21
	(2.66)	(3.33)	(1.50)	(2.93)	(3.86)	(4.69)	(4.63)	(4.61)
GDP60	−0.44	−0.52	−0.48	−0.58	−0.38	−0.38	−0.39	−0.38
	(−3.28)	(−3.17)	(−3.37)	(−3.47)	(−3.61)	(−3.25)	(−3.06)	(−2.95)
PRIM60	3.26	2.85	3.98	3.70	3.85	2.66	2.62	2.65
	(3.38)	(2.43)	(4.66)	(3.72)	(4.88)	(2.66)	(2.53)	(2.56)
GINI60	−5.70	−15.98	3.58	−12.93		−3.47	−3.45	−3.47
	(−2.46)	(−3.21)	(−1.81)	(−3.12)		(−1.82)	(−1.79)	(−1.80)
GINILND					−5.50	−5.23	−5.24	−5.21
					(−5.24)	(−4.38)	(−4.32)	(−4.19)
DEMOC* GINILND							0.12 (0.12)	
DEMOC								0.02 (0.05)
$\overline{R}^2$	0.28	0.27	0.25	0.26	0.53	0.53	0.51	0.51

The dependent variable is average per capita growth rate over 1960–1985. t-statistics are in parentheses. Independent variables are defined as follows:

GDP60:	Per capita GDP level in 1960
PRIM60:	Primary school enrollment ratio in 1960
GINI60:	Gini coefficient of income inequality, measured close to 1960 (see Appendix for dates)
GINILND:	Gini coefficient of land distribution inequality, measured close to 1960 (see Appendix for dates)
DEMOC:	Democracy dummy.

Two-stage least squares regressions use GDP60, PRIM60, literacy rate in 1960, infant mortality in 1965, secondary enrollment in 1960, fertility in 1965, and an Africa dummy as instruments.

For the high quality sample the coefficients of $GINI60$ i.e. the Gini-coefficient of 1960 is always significant on a 5% level and are always negative. Therefore the regression suggests a negative link between economic growth and inequality. Alesina and Rodrik (1994) also added a Gini-coefficient for land distribution($GINILND$) to analyze the impact of other types of inequality. For our purpose, this is not relevant.

Repeating the same procedure for the shorter time horizon (1970-1985) leads to the results listed in table 5

Tab. 5: GROWTH REGRESSION of Alesina and Rodrik(1994)

	High-quality sample (N = 46)	Largest possible sample (N = 70)	Largest possible sample (N = 49)	Largest possible sample (N = 41)		
	OLS (9)	OLS (10)	OLS (11)	OLS (12)	OLS (13)	OLS (14)
Const.	4.56 (2.67)	2.80 (2.00)	4.88 (3.16)	7.22 (3.79)	7.18 (3.69)	7.22 (3.74)
GDP70	−0.29 (−2.60)	−0.27 (−2.33)	−0.21 (−2.09)	−0.28 (−2.58)	−0.28 (−2.23)	−0.27 (−2.15)
PRIM70	3.28 (2.46)	3.79 (3.52)	3.45 (2.65)	2.77 (1.83)	2.81 (1.79)	2.81 (1.80)
GINI70	−9.71 (−3.62)	−7.95 (−3.49)		−5.71 (−2.33)	−5.74 (−2.30)	−5.73 (−2.30)
GINILND			−8.14 (−5.49)	−6.41 (−3.79)	−6.39 (−3.69)	−6.46 (−3.71)
DEMOC* GINILND					−0.11 (−0.13)	
DEMOC						−0.09 (−0.15)
$\overline{R}^2$	0.28	0.23	0.43	0.46	0.45	0.45

The dependent variable is average per capita growth rate over 1970–1985. t-statistics are in parentheses. Independent variables are defined as follows:

GDP70:	Per capita GDP level in 1970
PRIM70:	Primary school enrollment ratio in 1970
GINI70:	Gini coefficient of income inequality, measured close to 1970 (see Appendix for dates)
GINILND:	Gini coefficient of land distribution inequality, measured close to 1960 (see Appendix for dates)
DEMOC:	Democracy dummy.

Source: Alesina, A., & Rodrik, D. (1994). Distributive Politics and Economic Growth. Quarterly Journal of Economics 109 , S. 465-490.

This adjustment of the data set improves the results. All coefficients on $GINI60$ are now highly significant. The other results are in line in sign with other studies(Persson and Tabellini, 1994) and the magnitude of the coefficients is approximately the same as in table 4.

The empirical findings of Alesina and Rodrik (1994) and Persson and

Tabellini (1994) - although based on a different theoretical reasoning - support a negative effect of inequality on growth. While Persson and Tabellini (1994) argue that higher inequality leads to redistribution and therefore decreases incentives, Alesina and Rodrik (1994) argue that higher inequality leads to redistribution and therefore to market distortions. In any case, their cross-country studies reveal that higher inequality tends to harm growth opening a new discussion on the validity of the methods used.

3 Panel Data Studies

As shown in the previous section, most of the cross-country regressions result in a negative relation between inequality and growth. But in the late 1990's new research was based on panel data. So the question arises: What is the problem with cross-country estimations in general?

- It can be shown, that many of the coefficients in these studies are not robust. This means that by adding variables, the coefficient becomes insignificant.
- There are measurement errors in inequality and a bias due to omitted variables.
- The interpretation of cross-country regressions is that countries with lower inequality tend to grow faster in the long-run, but they do not focus on how a change in inequality within a country changes this country's growth rate. This is the main focus of panel regression.

Measurement errors occur in cross-country data mainly because different definition of key variables and because of inaccuracy in the data. Usually this measurement error is quite high for inequality. Deininger and Squire(1996) came up with a new data set on inequality, that tries to minimize this measurement error. Data that

- were not from household surveys
- were not take from a representative sample and
- did not include wage, non-wage earnings and none monetary income

were excluded. The Deininger and Squire(1996) study was not only a breakthrough because it uses high quality data, but also because it has a time-series dimension for enough countries that allows to perform panel regression.

Deininger and Squire(1998) performed a cross-country regression on that data set that brought again a negative relation between inequality and growth. But when they added regional dummies to their growth regression, the results became highly insignificant. This is usually a sign of omitted variables especially because the regional dummies are significant, meaning that regional effects on growth are not captured by the model. The general approach to reduce this bias is to use panel data because it controls for country characteristics that are constant over time.

The main focus in this section will be on two papers of Forbes(2000) and Barro(2000), both using panel data and the inequality data set of Deininger and Squire(1996).

In the 1970's, the Kuznets Curve was seen as an empirical regularity even though it only explains little of the variance. Barro(2000) uses an extended version of the neoclassical growth model for his empirical analysis to show that this relationship has not weakened over time, as suggested by other papers in the 1990's.

In this neoclassical environment, the growth rate of per capita output g is a function of the current level of per capita output y and the long-run level of per capita output y^*.

$$g = f(y, y^*)$$

Assuming that y is given, an increase in y^* increases g. This increase in the long-run level can be caused by various reasons such as an enforcement in property rights or an increase in political stability.

Using a sample from 1960 to 1995 for more than 100 countries, Barro mentioned that it might be difficult to measure the variables in a consistent way across countries over time since measurement errors in underdeveloped countries are quite common. Also causality problems might arise since we are looking at the effects of e.g. government policies on economic growth, but in reality, the policies are often reactions to the economic environment. The sample uses the average growth rates over three decades, 1965-1975, 1975-1985 and 1985-1995 because some of the variables can not be determined in years. The results of the general growth regression estimated with random effects are shown in Table 6.

Tab. 6: GROWTH REGRESSION of Barro(2000)

Independent Variable	Estimated Coefficient in Full Sample	Estimated Coefficient in Gini Sample
log(per capita GDP)	0.123 (0.027)	0.101 (0.030)
log(per capita GDP) squared	−0.0095 (0.0018)	−0.0081 (0.0019)
Government consumption/GDP	−0.149 (0.023)	−0.153 (0.027)
Rule-of-law index	0.0173 (0.0053)	0.0103 (0.0064)
Democracy index	0.053 (0.029)	0.041 (0.033)
Democracy index squared	−0.047 (0.026)	−0.036 (0.028)
Inflation rate	−0.037 (0.010)	−0.014 (0.009)
Years of schooling	0.0072 (0.0017)	0.0066 (0.0017)
log(total fertility rate)	−0.0250 (0.0047)	−0.0303 (0.0054)
Investment/GDP	0.059 (0.022)	0.062 (0.022)
Growth rate of terms of trade	0.164 (0.028)	0.122 (0.035)
Numbers of observations	79, 87, 84	39, 56, 51
R^2	0.67, 0.49, 0.41	0.73, 0.62, 0.60

Source: Barro, R. (2000). Inequality and Growth in a Panel of Countries. Journal of Economic Growth , pp. 5-32.

The data shows a positive relation with the log of per capita GDP, the rule-of-law index, the democracy index, the years of schooling, the investment ratio and with the growth rate of terms of trade. A negative relation can be seen for the log of per capita GDP squared, government consumption as a share of GDP, the democracy index squared, the inflation rate and the log of the total fertility rate. We get different R^2 for each of the three centuries.

The question arises, what measurement of inequality one should add to the growth equation. Usually there are two different methods: either taking the Gini-coefficient or using quintiles of the income distribution. Barro(2000) concentrates on the Gini coefficient in his analysis. Since the data for inequality measures are rare some of the observations are lost.

Adding the Gini-coefficient to the growth regression in table 6 leads to the results of table 7.

Tab. 7: The effects of Gini on growth rates (Barro,2000)

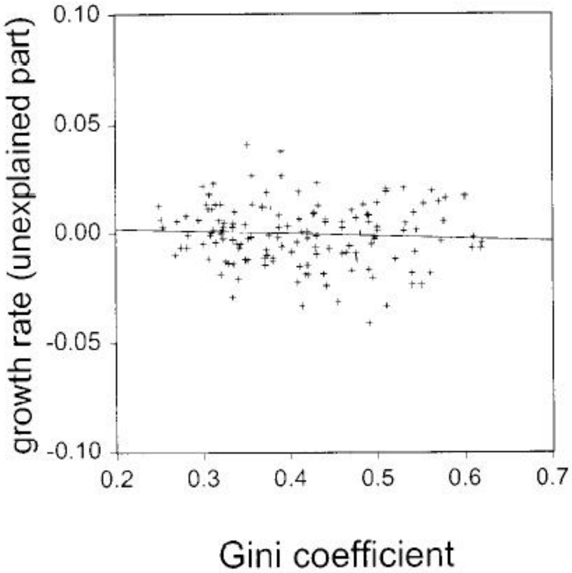
	Gini	Gini* log(GDP)	Gini (low GDP)	Gini (high GDP)	Wald Tests (p-values)
Growth rate regressions					
	0.000				
	(0.018)				
	−0.328	0.043			0.061
	(0.140)	(0.018)			
			−0.033	0.054	0.011,
			(0.021)	(0.025)	0.003*
Fertility variable omitted					
	−0.036				
	(0.017)				
	−0.364	0.043			0.014
	(0.155)	(0.020)			
			−0.059	−0.003	0.017,
			(0.021)	(0.026)	0.078*

Source: Barro, R. (2000). Inequality and Growth in a Panel of Countries. Journal of Economic Growth , pp. 5-32.

Column 1 shows that the parameter of *Gini* is close to 0. This implies that there is a almost zero-relationship between inequality and growth as shown in figure 4. However, if we exclude the fertility-rate which is usually correlated with inequality[1] the coefficient of Gini becomes significantly negative. A one standard-deviation increase in the Gini-coefficient would decrease the growth rate by 0.4%. This magnitude is the same as in earlier studies but it seems that it is only a proxy for the fertility rate.

[1] Children are often seen as a retirement provision in the low income groups of developing countries. Therefore the high fertility rate in this group will enlarge the group compared to the high income group and therefore leading to more inequality.

Fig. 3: GROWTH VS. GINI (Barro, 2000)



Source: Barro, R. (2000). Inequality and Growth in a Panel of Countries. Journal of Economic Growth , pp. 5-32.

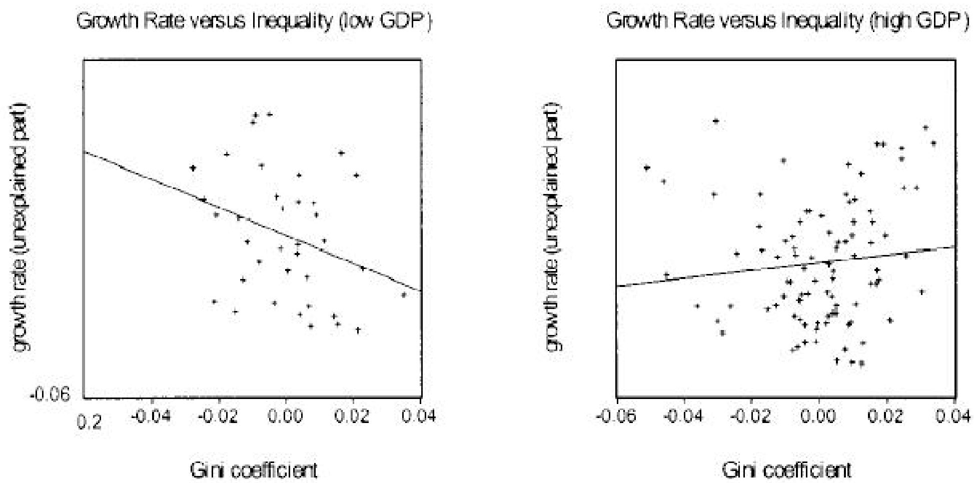
Even more interesting is the result in column two of Barro's growth rate regression. Adding an interaction term $Gini * log(GDP)$ yields to significant results for both coefficients, individually and jointly. By simple calculations we can now determine the critical GDP level that changes the impact of inequality on growth.

$$\begin{aligned} \gamma &= -0.328\,Gini + 0.043\,Gini\,log(GDP) \\ \frac{\partial\gamma}{\partial Gini} &: -0.328 + 0.043\,log(GDP) = 0 \\ \Rightarrow GDP &= e^{\frac{0.328}{0.043}} \approx 2070\$ \end{aligned}$$

Each level above 2070$ (of the year 1985) leads to a positive relation between growth and income inequality cet. par.. The higher the GDP-level, the higher the growth rate cet. par. since the coefficient of the cross-term is positive. For levels below 2070$ the relation will be negative.

Dividing the sample as suggested by adding an interaction term, we get two separate equations, one for countries with low GDP-levels (below 2070$) and one for countries with high GDP-levels (above 2070$). The estimation output is shown in column 3 in table 7. The result is basically the same as in the equation with the interaction term. Figure 4 shows the this relationship for low and high GDP per capita countries.

Fig. 4: GROWTH VS. GINI divided sample (Barro, 2000)



Source: Barro, R. (2000). Inequality and Growth in a Panel of Countries. Journal of Economic Growth , pp. 5-32.

Going back to the empirical findings of Kuznets, one should see an inverted U-shaped relationship between inequality and the level of GDP per capita. Using again the Gini coefficient as a measure for inequality and the logarithm of the GDP per capita, we estimate the following equation:

$$Gini_{i,t} = \beta_1 * log(GDP_{i,t}) + \beta_2 * log(GDP_{i,t})^2 + \epsilon_{i,t} \quad (4)$$

where i is the country index and t is the time period and $\epsilon_{i,t}$ the error term.

Tab. 8: DETERMINANTS OF INEQUALITY(random effects) (Barro, 2000)

Variable						
log(GDP)	0.407	0.407	0.437	0.415	0.443	0.401
	(0.090)	(0.081)	(0.078)	(0.084)	(0.074)	(0.073)
log(GDP) squared	−0.0275	−0.0251	−0.0264	−0.0254	−0.0280	−0.0253
	(0.0056)	(0.0051)	(0.0049)	(0.0053)	(0.0047)	(0.0046)
Dummy: net	—	−0.0493	−0.0480	−0.0496	−0.0515	−0.0517
income or spending		(0.0094)	(0.0087)	(0.0094)	(0.0090)	(0.0087)
Dummy: individual	—	−0.0134	−0.0143	−0.0119	−0.0146	−0.0146
vs. household data		(0.0086)	(0.0080)	(0.0087)	(0.0084)	(0.0082)
Primary schooling	—	−0.0147	−0.0152	−0.0161	−0.0065	−0.0094
		(0.0037)	(0.0036)	(0.0037)	(0.0039)	(0.0038)
Secondary schooling	—	−0.0108	−0.0061	−0.0109	−0.0176	−0.0174
		(0.0070)	(0.0070)	(0.0070)	(0.0067)	(0.0065)
Higher schooling	—	0.081	0.072	0.082	0.099	0.097
		(0.034)	(0.032)	(0.034)	(0.032)	(0.032)
Dummy: Africa	—	0.113	0.135	0.113	0.116	0.115
		(0.015)	(0.016)	(0.015)	(0.014)	(0.014)
Dummy: Latin America	—	0.094	0.089	0.092	0.095	0.095
		(0.012)	(0.012)	(0.012)	(0.012)	(0.011)
Rule-of-law index	—	—	−0.040	—	—	—
			(0.019)			
Democracy index	—	—	—	−0.003	—	—
				(0.015)		
Openness	—	—	—	—	0.050	0.422
					(0.013)	(0.113)
Openness*log(GDP)	—	—	—	—	—	−0.0445
						(0.0133)
Number of observations	49, 61	40, 59	40, 57	35, 59	38, 55	38, 55
	68, 76	61, 70	56, 67	61, 70	57, 64	57, 64
R-squared	0.12, 0.15	0.52, 0.59	0.50, 0.58	0.56, 0.59	0.61, 0.64	0.63, 0.63
	0.18, 0.22	0.67, 0.67	0.78, 0.72	0.67, 0.67	0.69, 0.72	0.74, 0.74

Source: Barro, R. (2000). Inequality and Growth in a Panel of Countries. Journal of Economic Growth , pp. 5-32.

The results are striking. Both coefficients, the linear and the quadratic one, are significant: the Gini coefficient increases up to a GDP-level of 1636$(of the year 1985) and starts to decrease afterwards. As we can see, the explained variation is between 0.12 and 0.22 and therefore really small but this is also in line with other research form recent years. Each R^2 stands for the explained variation in each of the four panel periods. From a theoretical point of view, it makes sense that economic development alone only explains a small fraction of the variation in inequality. Adding control variables as shown in row 2, 3 and 4 of table 8 increases the fit of the regression, but leaves the interesting coefficients almost unchanged.

Generally speaking, the overall data set shows no relation between inequality and growth. But there is evidence that inequality increases growth in poor countries, while it decreases growth in richer ones. Besides the Kuznets curve can be seen as a empirical regularity, even though it only explains very little of the variance in inequality.

Another research paper by Forbes(2000) chooses a growth model that is often used in the literature:

$$\begin{aligned} Growth_{i,t} &= \beta_1 * inequality_{i,t-1} \\ &\quad +\beta_2 * income_{i,t-1} \\ &\quad +\beta_3 * maleEducation_{i,t-1} \\ &\quad +\beta_4 * femaleEducation_{i,t-1} \\ &\quad +\beta_5 * PPPI_{i,t-1} + \alpha_i + \delta_t + \epsilon_{i,t} \end{aligned} \tag{5}$$

where i is the country index and t is the time period, α_i are country dummies and δ_t period dummies and $\epsilon_{i,t}$ is the error term. The sample ranges from 1966 to 1995 and is divided in six 5-year periods. *Growth* is the average growth rate of real GNP per capita over the 5 years, *inequality* is taken from the Deininger and Squire(1996) data set and is measured by the Gini-coefficient, *income* is measured by the log of the real GNP per capita, *maleEducation* and *femaleEducation* are average years of secondary schooling and $PPPI$ represents the market distortions that are approximated by the price level of investment.

The usually used fixed effect or random effect technique is under assumption of equation 5 not optimal because of the lagged income. Chamberlain's π-matrix approach is also rejected by several test. Therefore the method introduced by Arellano and Bond(1991) is used. It corrects for the bias introduced by the lagged endogenous variable and permits a certain degree of endogeneity in the other regressors.

Tab. 9: REGRESSION RESULTS of Forbes(2000)

Estimation method	Five-year periods: Fixed effects (1)	Random effects (2)	Chamberlain's π-matrix (3)	Arellano and Bond (4)	Ten-year periods: fixed effects (5)
Inequality	0.0036	0.0013	0.0016	0.0013	0.0013
	(0.0015)	(0.0006)	(0.0002)	(0.0006)	(0.0011)
Income	−0.076	0.017	−0.027	−0.047	−0.071
	(0.020)	(0.006)	(0.004)	(0.008)	(0.016)
Male Education	−0.014	0.047	0.018	−0.008	−0.002
	(0.031)	(0.015)	(0.010)	(0.022)	(0.028)
Female Education	0.070	−0.038	0.054	0.074	0.031
	(0.032)	(0.016)	(0.006)	(0.018)	(0.030)
PPP	−0.0008	−0.0009	−0.0013	−0.0013	−0.0003
	(0.0003)	(0.0002)	(0.0000)	(0.0001)	(0.0003)
R^2	0.67	0.49			0.71
Countries	45	45	45	45	45
Observations	180	180	135	135	112
Period	1965–1995[a]	1965–1995[a]	1970–1995	1970–1995	1965–1995

Notes: Dependent variable is average annual per capita growth. Standard errors are in parentheses. R^2 is the within-R^2 for fixed effects and the overall-R^2 for random effects.

[a] Estimates are virtually identical for the period 1970–1995 (with 135 observations).

Source: Forbes, K. (2000). A Reassessment of the Relationship between Inequality and Growth. American Economic Review 90 , pp. 869-887.

In table 9 the results of all the different methods are listed. Forbes(2000) shows that the coefficients estimated by the Arellano and Bond method are consistent and efficient. All coefficients are highly significant and do not differ from those found in other growth literature. The exception is the coefficient of inequality. It turned out, that it is always significant and positive, no matter which method is used. This contradicts to most of the research done so far. Also the magnitude of the coefficient is surprisingly high. A 0.1 increase in the Gini-coefficient will increase the average annual growth rate by 1.3% over the next period.

This does not mean that previous research was totally wrong. One has to distinguish the main interest of the cross-section studies and the Forbes paper and how to interpret the results. While cross-section results analyze the differences in the relation of inequality and growth across countries, the coefficients estimated by Forbes(2000) have to be interpreted as how the change in inequality effects the growth rate within each country across time.

This result does not necessarily lead to the conclusion that cross-country regression analysis is wrong. The panel method measures basically a short-run to medium-run relation of inequality and growth since a 5-year period is used. Cross-country regressions are usually based on 25- to 30-year -average-observations, meaning that they measures more the long-term relation between inequality and growth. Therefore it might be interesting to enlarge the panels to ten years of observations. The long-run coefficient of the Gini-coefficient of the panel regression stated in table 9 column 5 still shows the positive sign but becomes insignificant. The results can not be interpreted easily since the data used is restricted due to low degrees of freedom. So to get a better understanding of this long-run relation of inequality and growth within a country, more data has to be available for a longer time.

An interesting result arises by testing the robustness of the coefficients. In general the positive coefficients of $Inequality$ seem to be strongly robust. Dividing the data set into poor and rich countries, the relation between inequality and growth stays positive in each of the groups. This is exactly the opposite result, that Barro(2000) shows in his panel analysis. But there might be a simple reason for the contradiction. Barro added poor countries to his panel data while the Forbes analysis contains almost no very poor developing countries, since the data for these countries are limited.

Forbes(2000) estimates models that are used in the literature and uses different regression methods for the different model. The models show negative relation between inequality and growth when using cross-country analysis but when using panel technique the results change to a positive relation.

However the result of Forbes(2000) are in general not a contradiction to the findings of cross-country studies mentioned in section 2 since the results of panel techniques seem to be not reliable as far as the long-run is concerned. Also the findings of Barro(2000) are not rejected and point out the weakness of the Forbes(2000) paper. The data used are not representative since very poor countries are missing and this may bias the coefficients. On the other hand, Barro(2000) uses low-quality inequality data of poor countries that may also influence the results. But in general, the results for rich countries are the same. Higher inequality increases the growth rate within a country in the short-run. But both authors mentioned that the relation between inequality and growth is still far from being solved. More research has to be done in this field and also more data have to be collected especially for developing countries.

4 Conclusion and Criticism

The analysis of the relation between income inequality and economic growth seems to be an empirical mystery. Summarizing the empirical findings, the literature that exploits the cross-section variation (as the random effects method used in Barro(2000) and all the cross-sectional studies) suggests a negative pattern, the panel-data approach that usually covers the time-series variation yields a positive relation. This subsection will first present a paper of Halter, Oechslin and Zweimüller(2010) that tries to answer the variety of empirical results. The second part can then be seen as a criticism of the methods used in the analysis and additionally it points out new possible fields for research.

4.1 Concluding remarks

How can we explain these distinct results in the cross section analysis and the panel analysis? Forbes(2000) pointed already out that the two approaches usually have a different meaning. While the cross section studies focus on long-run growth, the approach focusing on time-series takes a closer look on the short and medium-run. This argument is carried out in a paper of Halter, Oechslin and Zweimüller(2010). It provides a strong thought for the distinct results of the empirical analysis:

What explains positive short-run effects of inequality on growth? Kuznets (1955) already argued with the accumulation of savings, as we saw in chapter 1. Rosenzweig and Binswanger (1993) state that under the assumption of weak capital markets the realization of money-demanding projects can only be implemented under the condition of high inequality. The long-run negative effect of inequality and growth is theoretically driven by social and political actions and frameworks. Benabou (1996) argues that the higher inequality leads to higher political instability and therefore decreases growth. Perotti (1993) attributes the negative relation to expensive fiscal policy. Galor and Zeira (1993) trace it back to the reduction of human capital formation. Alesina and Rodrik (1994) argue that inequality harms growth by leading to higher redistribution and therefore distort markets. As we can see already, the theoretical negative effects of income inequality on growth are closely linked to the institutional and political framework. Usually political and institutional changes take a lot of time and therefore are only valid for the long-run while the positive effects are already present in the short-run, since they are part of economic mechanisms. If these positive short-run effects dominate the negative long-run effects, the overall long-run effect on the growth rate can be positive. But obviously it can also be negative if they do not.

For the empirical analysis Halter, Oechslin and Zweimüller (2010) use the same model as Forbes (2000) (equation (5)) and the same method using first-difference GMM. The data is also quite similar, but the range is now from 1966-2005 and some countries are added. In addition, they use the system GMM to subtract long-run implications. The results of the first difference GMM(that extracts the overall short-term pattern) shown in table 10 are in line with the results presented by Forbes (2000) - in sign and approximately in magnitude.

Using the system GMM(that extracts only the long-term pattern) shown in table 11, the results are not that obvious.

The regression on the full sample leads to an insignificant negative coefficient of the Gini-coefficient. Splitting the sample in high income, high middle income and low-middle and low-income countries leads to a significant result in two of them. Lower inequality leads to lower growth in high

Tab. 10: GROWTH REGRESSION(first-difference GMM) of Halter, Oechslin and Zweimüller(2010)

	5-year growth rate of the real GDP p.c. (5-year periods)					10-year growth rate of the real GDP p.c. (10-year periods)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Forbes countries / periods	Forbes countries / more periods	Full sample	Full sample / high and up-mid	Full sample / low-mid and low	Full sample / high and up-mid	Full sample / low-mid and low
$\log(\text{GDP})_{t-1}$	-0.1236 *(0.09)	-0.1758 ***(0.002)	-0.1726 ***(0.005)	-0.1952 ***(0)	-0.1756 **(0.014)	-0.1988 (0.103)	-0.3879 *(0.001)
Gini coefficient	0.0075 ***(0.009)	0.0055 **(0.033)	0.0036 *(0.088)	0.0041 *(0.089)	0.0034 **(0.047)	0.0114 *(0.084)	0.128 (0.127)
Male schooling (yrs.)	-0.023 (0.787)	-0.0496 (0.54)	-0.036 (0.674)	-0.0905 (0.121)	0.1336 (0.159)	-0.0876 (0.542)	-0.0052 (0.969)
Female schooling (yrs.)	0.0766 (0.346)	0.027 (0.697)	0.028 (0.729)	0.0848 **(0.031)	-0.0265 (0.646)	0.016 (0.861)	-0.0099 (0.942)
Price level of inv.	-0.0013 **(0.011)	-0.0008 (0.151)	-0.0005 (0.317)	-0.006 (0.113)	-0.0011 **(0.082)	0.0001 (0.938)	-0.0021 (0.236)
Number of countries	42	42	70	37	33	30	14
Number of obs.	131	225	273	180	93	68	28
Number of inst.	75	140	140	140	93	30	26
M1	-1.96	-3.38	-3.44	-2.72	-2.14	-2.98	-1.67
M2	1.67	-0.89	-0.73	-1.41	-1.22	-0.025	-1.45
Hansen	1	1	1	1	1	0.289	0.943

All regressions include period dummies; ***, **, * denote significance at the 1, 5, 10% levels, respectively; *p*-values in parentheses; M1 and M2 are the *t*-values of the tests for, respectively, first-order and second-order serial correlation in the differenced error terms; Hansen denotes the *p*-value of the Hansen test of over identifying restrictions.

Source: Halter, D., Oechselin, M., & Zweimüller, J. (2010). Inequality and Growth: The Neglected Time Dimension. Working Paper Institute for Empirical Research in Economics University of Zurich .

income countries while the adverse relation appears in the low-middle and low income group. This is perfectly in line with Barro(2000), Persson and Tabellini(1994) and Rodrik and Alesina(1994). For countries that are not among the richest, the long-term effect of higher inequality on growth is negative.

4.2 Criticism

The empirical analysis pointed out so far should not be seen as exactly brilliant. Criticism of the research done so far was stated in a paper from Voitchovsky(2005). While former research was always based on a single inequality statistic, the paper suggests to use more than one inequality measures for a simple reason: From the theoretical point of view, the positive and negative effects of inequality on economic growth can not only be divided in long-run and short-run effects but can also be split by the distribution itself. While inequality at the top end of the income distribution is positively linked to growth, higher inequality at the bottom end of the income distribution will theoretically lead to a decrease of growth. Using the Gini-coefficient as a source of inequality only takes the average of these effects into account.

Tab. 11: GROWTH REGRESSION(system GMM) of Halter, Oechslin and Zweimüller(2010)

	5-year growth rate of the real GDP p.c. (5-year periods)				10-year growth rate of the real GDP p.c. (10-year periods)	
	(1)	(2)	(3)	(4)	(6)	(7)
	Full sample	Full sample / high	Full sample / up-mid	Full sample / low-mid and low	Full sample / high	Full sample / low-mid and low
$\log(GDP)_{t-1}$	-0.0047 (0.691)	-0.0346 (0.281)	-0.1474 ***(0.000)	-0.037 *(0.099)	-0.1648 ***(0.002)	-0.0488 (0.283)
Gini coefficient	-0.0013 (0.191)	0.0021 **(0.011)	0.0006 (0.55)	-0.0049 **(0.02)	0.0058 **(0.014)	-0.0103 **(0.028)
Male schooling (yrs.)	0.0907 ***(0.007)	0.0299 (0.121)	0.0218 (0.553)	0.0029 (0.946)	0.0761 *(0.1)	-0.0409 (0.746)
Female schooling (yrs.)	-0.0782 **(0.034)	-0.0151 (0.437)	-0.028 (0.407)	0.0481 (0.345)	-0.0467 (0.32)	0.1156 (0.275)
Price level of inv.	-0.0014 ***(0.002)	-0.0013 ***(0.004)	-0.0016 *** (0.001)	-0.0006 (0.372)	-0.0019 **(0.044)	-0.0022 (0.218)
Number of countries	90	26	14	50	24	42
Number of obs.	404	154	79	171	71	76
Number of inst.	176	150	77	129	46	42
M1	-4.08	-2.53	-2.16	-2.56	-2.53	-1.1
M2	-1.27	-0.08	-1.55	-0.94	-1.01	-1.57
Hansen	1	1	1	1	0.994	0.952

All regressions include period dummies; ***, **, * denote significance at the 1, 5, 10% levels, respectively; *p*-values in parentheses; M1 and M2 are the *t*-values of the tests for, respectively, first-order and second-order serial correlation in the differenced error terms; Hansen denotes the *p*-value of the Hansen test of over identifying restrictions.

Source: Halter, D., Oechselin, M., & Zweimüller, J. (2010). Inequality and Growth: The Neglected Time Dimension. Working Paper Institute for Empirical Research in Economics University of Zurich .

Using again the arguments of an marginal saving rate that increases with the level of income(Kuznets, 1955 and others), the positive effect concerning the inequality at the top end distribution seems reasonable. Higher poverty reflected by higher income inequality at the bottom end distribution usually increases anti-social behavior, increases risk and therefore reduces growth. Higher overall inequality usually leads to instability and therefore to lower growth (Alesina and Perotti, 1996). The effect of taxes is uncertain. While increased taxation has a negative effect on people at the top end distribution, the effect on people at the bottom end distribution is positive because productivity increases by relaxing the credit constraints(Benabou, 1996). Generally speaking, most positive effects can be associated with the inequality in the top end while most negative effects can be linked to the inequality in the bottom end distribution.

Voitchovsky(2005) uses data from the Luxembourg Income Study (LIS). The inequality is measured by the ratio of income percentiles, e. g. the top end ratio as 90/75 and the bottom end as 50/10. Data is available for 21 countries from 1975 till 2000. The used sample contains only wealthy democratic countries and it has to be mentioned that it is by far smaller than most of the samples used in other research. It can be already seen in the data that it is possible that the Gini coefficient stays constant over time while the top and bottom end inequality changes noticeably(e. g. the UK in the 1970's).

The model conforms to other research by using the same 5 year structure in the model (as Forbes, 2000). Facing also the same difficulties, the best estimation technique suggested is the system GMM. The results are stated in table 12:

Tab. 12: GROWTH REGRESSION of Voitchovsky(2005)

Sample	Baseline sample			Whole sample		
Col no.	1	2	3	4	5	6
Instrument set	b	b	b	default	default	default
y_{t-1}	−0.2627***	−0.2315***	−0.2379***	−0.2998***	−0.2785***	−0.2841***
	(0.0771)	(0.0722)	(0.0661)	(0.0822)	(0.0820)	(0.0725)
Invest_t	0.0166***	0.0138**	0.0146***	0.0304***	0.0279***	0.0261***
	(0.0069)	(0.0075)	(0.0074)	(0.0124)	(0.0120)	(0.0111)
AvgYrsSch_{t-1}	0.0427***	0.0464***	0.0571***	0.0447***	0.0535***	0.0587***
	(0.0210)	(0.0207)	(0.0220)	(0.0222)	(0.0232)	(0.0257)
Gini_{t-1}	−0.1836	−1.3885**		−0.1418	−1.5899***	
	(0.5649)	(0.7535)		(0.5464)	(0.7597)	
$90/75_{t-1}$		1.0168**	0.6932*		1.1110***	0.6823
		(0.5231)	(0.4698)		(0.5527)	(0.5180)
$50/10_{t-1}$			−0.1628***			−0.1566***
			(0.0799)			(0.0777)
p-value[1]	0.745	0.109	0.120	0.795	0.088	0.123
m1	−2.249	−2.090	−2.131	−1.875	−1.783	−1.955
m2	−0.792	−0.835	−0.812	−0.568	−0.591	−0.602

21 countries in baseline sample and 81 obs.; 25 countries in whole sample and 89 obs. Time dummies included; dummy for Eastern European countries added in whole sample analysis. First-step estimates reported. Robust standard errors in parenthesis. The dependent variable is Δy_t where $t-(t-1)$ is a 5-year period.

[1] Wald (joint) test on the inequality variable coefficient(s) in the regression.

***, **, * indicates that the coefficient is significantly different from 0 at the 5, 10 and 15 % significance levels, respectively.

Instrument set b: $\Delta 50/10_{t-1}$ added as an instrument for the equations in levels.

Source: Voitchovsky, S. (2005). Does the Profile of Income Inequality Matter for Economic Growth. Journal of Economic Growth , pp. 273-296.

Using only one of the measures of inequality (e. g. Gini coefficient, the 90/75 - ratio or the 50/10 ratio) leads to a insignificant negative coefficient for the Gini-coefficient and the bottom end ratio and to an insignificant positive coefficient for the top end ratio. But adding two of them (the Gini-coefficient and the top end ratio) to the growth regression leads to a highly significant result. Testing for joint significance (Wald-test) suggests that both are jointly significant. The economic interpretation of our coefficients is now the following: Once controlling for the top end inequality, the Gini-coefficient represents the inequality in the bottom end of the distribution. The effect of inequality in the top end of the distribution is positive, while the effect is adverse in the bottom end of the distribution. Sensitivity testing

(e. g. using different models, including dummies and so on) points out that the signs of the coefficients are robust.

The paper of Voitchovsky(2005) points out the limitation of using a single inequality statistic. The important question is whether the average effect of inequality on growth is interesting or whether the more complex relation within the income distribution is of interest. The overall effect on growth can not be determined in this model, since the magnitude of the coefficients vary with changing control variables and sample size. In order to be able to draw more conclusions about the relation between income inequality and economic growth, one needs to make further investigation. Additionally, better and bigger data sets would ease the problems faced so far.

Table 13 summarizes the main results of the empirical studies discussed so far. These results led to various theoretical models. Most of the models discussed in the next can explain parts of these empirical results. One should keep in mind that this research should not be seen as an explanation for the relationship between income inequality and economic growth in general but it points out that there is a relationship that might be different in the short run and the long run. Additionally, this relation also depends on how we define income inequality and how we measure it.

Tab. 13: Different studies on the relation of inequality and growth

	Method	Overall result	rich countries	poor countries	Remark
Persson and Tabellini (1994)	OLS	-	n. a.	n. a.	democracies vs. non-democracies
Alesina and Rodrik (1994)	OLS	-	n. a.	n. a.	Gini for land distribution
Barro (2000)	R. E.	0	+	-	low-quality data
Forbes (2000)	GMM	+	+	+	almost no developing countries
	CONCLUSION				
Halter, Oechslin and Zweimüller (2010)	GMM	**- in l.r. + in s.r.**	**+ in l.r. + in s.r.**	**- in l.r. + in s.r.**	long-run(l.r.) and short-run(s.r.) differences
	CRITICISM				
Voitchovsky (2005)	GMM	**(-)**	**n. a.**	**n. a.**	distinct inequality measures

( ) stands for insignificant results
n.a. stands for not applicable
0 stands for a zero relation

Part III. Theory

In general there are two approaches that try to explain the relation between inequality and growth:

- the political economy approach and
- the capital markets imperfections approach

While the first one is based on the political system that under higher inequality will produce market distortions and therefore decreases incentives for the upper part of the income distribution, the second approach usually leads to underinvestment in capital. The results of both theories are usually in line. They suggest that higher inequality harms growth. For each of the two approaches, a model will be presented in detail later in this chapter. But there are some exceptions that suggest that the relation between inequality and growth is ambiguous or even negative.

The main goal of this part is to give a brief overview over the recent literature and models that are used to explain the relation between inequality and growth. The division of the two groups mentioned before seems to be reasonable. Therefore section 5 will introduce some of the political economy models, while section 6 briefly presents some of the models that are based on capital market imperfections. Additionally, chapter 7 will introduce other models that are not directly linked to one of the two approaches.

5 The Political Economy Models

5.1 Introduction

The political economy models are, as the name suggests, based on the political system. Income inequality has always an impact on the political decisions. Politicians tend to implement policies that are most popular. An often used argument in these models is the median voter theorem[2]. This subsection will give a brief overview over the most cited models in the political economy approach. Subsection 2 will then show the model of Alesina and Rodrick(1994) in detail. The model of Li and Zou(1998) is based on the model of Alesina and Rodrik(1994) but leads under some distinct assumption to the opposite result. Subsection 4 will than summarize the main findings of the political economy approach.

[2] The median voter theorem goes back to Black(1948) and states that the vote-maximizing strategy for a politician is to choose the preferred policy of the median voter. (assuming that there are two politicians and individuals choose the policy that is closest to their individually preferred preference)

Alesina and Rodrik(1994) developed a endogenous growth model in which individuals are endowed with different capital/labor shares. They include a government service in the production function that is necessary for production. It can be seen as the institutional framework for production. This government service is financed by a tax on capital. Under these assumptions, they show that individuals almost always prefer a higher tax rate to the growth maximizing tax rate (only pure capitalists would choose exactly the growth maximizing tax rate). Additionally they point out that the higher the relative factor endowments of the median voter (that can be used as a proxy for inequality), the higher will be the preferred tax rate. Applying the median voter theorem this tax rate will be chosen by the politicians. A higher tax rate will decrease incentives to invest and therefore results in a negative effect of inequality on economic growth.

Li and Zou(1998) used the model of Alesina and Rodrik(1994) and showed that under the assumption that the public service will not be used for production but for consumption services, the outcome of the model is the reverse one. But the authors mentioned that in real world, government uses public services for both, production and consumption services. Therefore the relation of inequality and growth is ambiguous.

Persson and Tabellini(1994) invented also a political economy model based on an overlapping generation model that leads to the same result as the model of Alesina and Rodrik(1994). They use model where income is taxed only for redistributive reasons (not to provide a public good that is necessary for private production). The main result is that inequality harms growth because it leads to policies that do not allow individuals to take the whole advantage from investment. And also the empirical findings of the two papers (as shown in section 2) are the same.

5.2 Model of Alesina and Rodrik(1994)

The political economy approach is based on the idea that economic growth is forced by the accumulation of capital. The incentives to accumulate individually are obviously closely linked to the individual benefit. In a society with a high inequality redistributive policies are more likely. But these policies automatically lower incentives to accumulate and therefore reduce growth.

5.2.1 The Model

The main idea of this section is to develop a model that shows how the political process can effect the long-run growth. The simplified model is taken from Alesina and Rodrik(1994).

It is a endogenous growth model and based on two factors:

- aggregated labor endowment l which is not accumulated
- aggregated capital endowment k that accumulates and includes all types of capital: physical capital, human capital and proprietary technology

Growth is obtained by the expansion of the capital stock k which is indeed a result of individual saving.

The aggregated production function is assumed to be the following:

$$y = Ak^{\alpha}g^{1-\alpha}l^{1-\alpha} \qquad 0 < \alpha < 1 \tag{6}$$

A is a technological factor. There is a tax on capital income τ that is used for government services g. Private production requires thhese government services. It can be seen as the framework for economic actions. Assuming that the government budget is balanced we get:

$$g = \tau k \tag{7}$$

We can see that the tax is linear in the capital stock. It is not a progressive tax on capital. But assuming that wage income is relatively even distributed compared to capital the tax can be seen as a progressive tax. And therefore there is an incentive for capital poor voters to vote for a higher tax on capital.

Assuming perfect competition and the aggregated labor supply perfect inelastic ($l = 1$) yields to the usual FOC by differentiating (6) with respect to k and l and substituting (7):

$$r = \alpha A\tau^{1-\alpha} \tag{8}$$

$$w = (1-\alpha)A\tau^{1-\alpha}k \tag{9}$$

The marginal productivity of capital (r) is independent of the aggregated capital stock k. Both marginal productivities w and r are increasing in the tax rate since a higher tax-rate increases g for a given k. And the wage rate (w) is also increasing in the capital stock k.

The after-tax wage and capital income y^l and y^k are as follows:

$$y^k = (r-\tau)k \tag{10}$$

$$y^l = wl = w \qquad since\ l = 1 \tag{11}$$

The role of the tax-rate in the model can be seen best in equation (10) and (11):

- First, the tax has an influence on the capital income y^k. On the one hand, the tax rate τ is positively linked to r as mentioned in equation (8). On the other hand it directly effects the net income of capital in a negative way. So obviously the overall effect is not clear but it is clear that it effects the accumulation of capital.

- Second, the higher the tax rate the higher the wage income y^l since the tax rate is positively linked to w as shown in equation (9). A higher tax-rate leads to an higher spending in public services g which increases productivity.

The relative factor endowment of individuals is assumed to be different. Each individual i is indexed by its relative factor endowment σ^i:

$$\sigma^i = \frac{l^i}{\frac{k^i}{k}} \tag{12}$$

meaning that an individual with low relative factor endowment is capital rich and vica versa. The individual net income y^i is given by the individual labor and capital stock:

$$\begin{aligned} y^i &= wl^i + (r-\tau)k^i = \eta\sigma^i k^i + (r-\tau)k^i \\ & \quad where\ \eta = (1-\alpha)A\tau^{1-\alpha} \end{aligned} \tag{13}$$

as we know that $\sigma^i * k^i = k * l^i$ from equation (12) . Note that the individual income is not only affected by the individual capital stock k^i but also by the aggregated capital stock k which is included in the wage rate w.

Assuming a logarithmic utility function, the consumption-saving decision can be stated as follows:

$$\begin{aligned} \max \quad & \int_0^\infty log\, c^i * e^{-\rho t} dt \\ s.t. \quad & \frac{dk^i}{dt} = y^i - c^i = \eta\sigma^i k^i + (r-\tau)k^i - c^i \end{aligned} \tag{14}$$

meaning that an individual maximizes its overall discounted utility (discount rate ρ) taking into account the change of the capital stock over time which is the difference of the individual income and what the individual consumes.

The solution of this problem is:

$$\frac{\frac{dc^i}{dt}}{c^i} = (r-\tau) - \rho \tag{15}$$

Calculations are shown in the Appendix in section A.

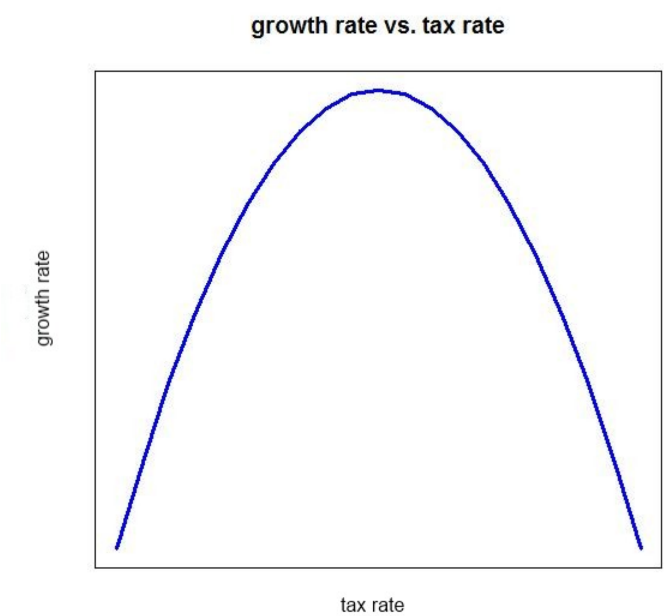
Assuming now that the tax rate is constant over time, individuals will accumulate along the steady-state path:

$$\frac{\frac{dc^i}{dt}}{c^i} = \frac{\frac{dk^i}{dt}}{k^i} = (r - \tau) - \rho \equiv \gamma(\tau) \tag{16}$$

meaning that there is a constant economy wide growth rate $\gamma(\tau)$, since individuals accumulate at the same rate. Individual factor endowment does not enter the solution. This also implicates that the relative factor endowments stay constant over time and therefore the wealth and income distribution is constant over time. The growth rate is a linear function of the difference of the after tax return on capital $(r - \tau)$ and the discount factor ρ. This result obviously eases the voting process since the median voter would change over time, meaning that also strategic voting hast to be taken into account.

Note that the overall effect of the capital tax on the solution is not linear. For high tax rates, the direct negative effect of the capital tax dominates the positive productivity enhancing effect (r depends positively on τ). For lower tax rates, the opposite is true. The relation between growth and the tax rate is an inverse U-shaped as can be seen in figure 5.

Fig. 5: The relation between growth rate and tax rate



The question arises, at which tax-rate τ^* the growth rate is maximized. Rewriting equation (16) and maximizing it leads to the following result:

$$\tau^* = [(1 - \alpha)\alpha A]^{1/\alpha} \tag{17}$$

Calculation can be seen in the Appendix in section A. The growth maximizing tax rate only depends on the technological factor A and does not depend on time.

It might be interesting to look at the preferred taxation of individual i. Therefore we look at the maximization problem of the government that is only concerned about individual i. The problem looks as follows:

$$\begin{aligned} \max \quad & \int log\, c^i * e^{-\rho t} dt \\ s.t. \quad & \frac{\frac{dk^i}{k^i}}{dt} = \gamma(\tau) = r - \tau - \rho \\ & \frac{\frac{dk}{k}}{dt} = \gamma(\tau) = r - \tau - \rho \\ & c^i = \left[\eta\sigma^i + \rho\right] k^i \end{aligned} \tag{18}$$

where the last constraint is derived from equation (14) and (16). The calculations are shown in the Appendix in section A. This yields to the following result:

$$\tau^i \left[1 - \alpha A(1-\alpha)(\tau^i)^{-\alpha}\right] = \rho(1-\alpha)\frac{\eta(\tau^i)\sigma^i}{\eta(\tau^i)\sigma^i + \rho} \tag{19}$$

$\frac{\eta(\tau^i)*\sigma^i}{\eta(\tau^i)*\sigma^i+\rho} = \theta^i$ is the share of labor income component in consumption expenditures of individual i. The calculations are shown in Appendix in section A. Time does not enter the expression of the best individual tax rate τ^i. Therefore τ^i is constant over time and depends positively on the relative factor endowment of individual i. One can show that the lower the relative factor endowment σ^i (the capital-richer the individual i), the lower the optimal individual tax rate τ^i. A pure capitalist (no labor income and therefore $\sigma^k = 0$) yields to an interesting result: $\tau^* = \tau^k = \left[(1-\alpha) * \alpha * A\right]^{1/\alpha}$

This implies that the tax rate that maximizes growth τ^* is the same as the pure capitalist would prefer. On a first glance, it might be surprising that the pure capitalist prefers some capital tax level to no taxes. But given the model assumptions that public service is needed for production the result becomes clear. Being endowed with only a small amount of wage income directly implicates that the individual would prefer a tax rate higher than the growth maximizing tax rate.

Assuming that wealth is evenly distributed among all individuals meaning that $\sigma^i = 1$ a government chooses a tax rate that is higher than the growth maximizing tax rate τ^*.

Recalling the equation of individual consumption $c^i = [\eta\sigma^i + \rho] k^i$ one can see that the consumption of a capitalist is $c^i = \rho * k^i$ and therefore independent of the tax rate. For all the other individuals the tax rate has also an effect on consumption by changing the wage income (see equation 11 and 9). A tax rate greater than τ^*

- increases the level of consumption and
- decreases the rate of growth of aggregated incomes and consumption

Assuming the tax rate to be at the growth maximizing level τ^*, a small increase will lead to a higher consumption level of all individuals except the pure capitalist and will decrease the growth rate of income and consumption slightly. The net effect will be beneficial for all except the pure capitalist.

Applying the median voter theorem in our model is valid. The preferences over a single issue(the tax rate) are single peaked and the relation between the ideal tax rate and the individual relative factor endowment is monotonic as we showed before. Therefore the chosen policy only depends on the preferred tax rate of the medium voter. This again depends on the relative factor endowment of the median voter. Since this tax rate is independent of time (as we have shown in equation 19) and the distribution of factor endowments do not change over time, it does not matter whether the voting takes place only at the beginning or in each period.

In a perfectly equal economy, the relative factor endowment of individuals is $\sigma^i = 1$ and therefore the relative income share of the median voter is $\sigma^m = 1$ the same. In real world $\sigma^m > 1$ meaning that the median voter is above the average share. Using this findings, we can use $\sigma^m - 1$ - the difference between the share of the median voter and the share of the average voter - as a measure of inequality. The higher $\sigma^m - 1$, the further away is the share of the median voter from the share of the average voter meaning that 50% of the population owns a very low share of the capital stock and therefore inequality is higher. This means the higher the relative income share of the median voter, the higher the inequality.

Since we are interested in income inequality, we have to rearrange equation (13) as follows:

$$y^i = \eta l^i k + (r - \tau)\frac{kl^i}{\sigma^i} = \left[\eta + \frac{(r-\tau)}{\sigma^i}\right] kl^i$$

Assuming that labor is unskilled, meaning that l^i is almost the same for all individuals, would imply an adverse relation of the income share σ^i and the individual income. The higher our gap of inequality in capital endowment, the larger will also be the gap between the average and the median income.

5.2.2 Summary

It is possible to summarize the model in 4 points that basically explain the model and the mechanisms that lead to the negative relation between inequality and economic growth:

- Firstly, the model points out that the preferred tax rate of all individuals is at least as high as the growth-maximizing tax rate (it would be the same only for a pure capitalists);
- secondly, the higher the relative factor endowment of the median voter, the higher will be the preferred tax rate of the median voter;
- thirdly, using the median voter theorem, the preferred tax rate of the median voter will be chosen by the government and
- fourthly, this implies that the higher the inequality, the higher the factor endowment of the median voter. The chosen tax rate will be therefore higher as well and therefore lowers the growth rate.

This leads to the overall result that the higher the inequality in wealth and also in income, the lower the growth rate.

Using the median voter theorem leads to interesting side result. Obviously the model seems to be more reliable for democracies than for dictatorships, since the voting process is only part of the political system of democracies. But still, even in a dictatorship the policy is based on social pressure. It also depends on the nature of the regime whether it prefers redistribution or tries to maximize growth.

5.2.3 Criticism

Several assumptions of the Alesina/Rodrik model might not reflect the whole story.For example, the inequality measure may be not adequate. There exist distributions where the difference between the median and the average voter is the same while inequality measured by e. g. the Gini-coefficient is totally different. Overall it might be a convenient and easy way to measure inequality but for the interpretation of the result one should take that into account.

The growth maximizing tax rate can not be achieved in practice since it would require the median voter to have no labor income at all. This is in real world not possible.

The assumption that labor is supplied perfectly inelastically is also a limiting one. In reality, the labor supply will not be totally unresponsive to changes of the real wage.

The tax regime used in the model is also not realistic. A tax on consumption and a tax on labor income may have different effects.

Assuming that government spending only effects production was criticized by Li and Zou(1998). They argue that government spending can be divided into production services and consumption services. Adding government spending to the individual consumption-saving decision the outcome is the inverse one, showing that inequality can also foster growth. The model specification will be shown in the next section.

5.3 The model of Li and Zou (1998)

The model of Li and Zou(1998) is based on the model of Alesina and Rodrik(1994) but the underlying framework is more general. Public spending can be divided in production services and consumption services. This model will focus on the consumption side of the government spending.

5.3.1 The Model

The individual utility-function U^i of the model is assumed to have the form of the CES-utility function and includes individual consumption c^i as well as public spending g:

$$U^i = \int_0^\infty \left[\frac{(c^i)^{1-\theta} - 1}{1-\theta} + \ln g \right] e^{-\rho t} dt \tag{20}$$

The production function is the same as in the model of Alesina and Rodrik(1994) (assuming that $\alpha = 1$):

$$y = Ak^\alpha g^{1-\alpha} = Ak \tag{21}$$

where k is the aggregated capital stock and A is a technological parameter. The price of capital is $r = \frac{dy}{dk} = A$. A positive tax rate on capital τ leads to an after-tax income of the individual $y^i = (1-\tau)Ak^i$. The net return is therefore $r(1-\tau) = A(1-\tau)$. The government spending (assuming a balanced budget in each period) will be:

$$g = \tau Ak \tag{22}$$

Individual capital accumulation is given by the difference of individual income and the individual consumption. Therefore $\frac{dk^i}{dt} = y^i - c^i = (1-\tau) * A * k^i - c^i$

The individual income share is:

$$\phi^i = \frac{Ak^i}{Ak} = \frac{k^i}{k}$$

which is also individual i's wealth (capital) share. Obviously individual with high ϕ^i is capital rich and therefore also income rich. Note that this definition is different to the definition in the model of Alesina and Rodrik(1994)!

The individuals maximization problem is therefore:

$$\begin{aligned} \max \quad & U^i = \int_0^\infty \left[\frac{\left(c^i\right)^{1-\theta} - 1}{1-\theta} + \ln g \right] e^{-\rho t} dt \\ s.t. \quad & \frac{dk^i}{dt} = y^i - c^i = (1-\tau)Ak^i - c^i \end{aligned}$$

The solution to this problem is shown in the Appendix in Part C and yields again to a growth rate that is the same for capital, income and consumption:

$$\frac{\frac{dc^i}{dt}}{c^i} = \frac{\frac{dk^i}{dt}}{k^i} = \frac{\frac{dy^i}{dt}}{y^i} = \frac{(1-\tau)A - \rho}{\theta} \equiv \gamma(\tau) \tag{23}$$

Since the growth rate is independent of any individual characteristics and not dependent on time, we can assume that the income shares do not change over time.

Again the maximization problem for a government that is only concerned about one individual (the median voter) is of interest. It can be stated as follows:

$$\begin{aligned} \max \quad & U^i = \int_0^\infty \left[\frac{\left(c^i\right)^{1-\theta} - 1}{1-\theta} + \ln g \right] e^{-\rho t} dt \\ s.t. \quad & \frac{\frac{dk^i}{dt}}{k^i} = \frac{(1-\tau)A - \rho}{\theta} \\ & \frac{\frac{dk}{dt}}{k} = \frac{(1-\tau)A - \rho}{\theta} \\ & \frac{dk^i}{dt} = y^i - c^i = (1-\tau)Ak^i - c^i \end{aligned}$$

The calculations for this maximization problem are shown in the Appendix in section C. It leads to the following FOC:

$$-A\left(\sigma^i k_0\right)^{1-\theta} \left(\frac{\rho - (1-\tau^i)(1-\theta)A}{\theta} \right)^{-\theta-1} + \frac{1}{\tau^i \rho} - \frac{A}{\theta \rho^2} = 0 \tag{24}$$

Assuming that the discounted utility has to be bounded, $\left(\frac{\rho - (1-\tau^i)(1-\theta)A}{\theta} \right) > 0$.

Taking the total differential will lead us to (for calculations see Appendix Part C):

$$\left[\frac{1-\theta^2}{\theta} A^2 \left(\sigma^i k_0\right)^{1-\theta} \left(\frac{\rho - (1-\tau^i)(1-\theta)A}{\theta} \right)^{-\theta-2} - \frac{1}{(\tau^i)^2 \rho} \right] d\tau^i + \tag{25}$$

$$\left[-A \left(\sigma^i k_0\right)^{-\theta} \left(\frac{\rho - (1-\tau^i)(1-\theta)A}{\theta} \right)^{-\theta-1} \frac{1-\theta}{\theta} k_0 \right] d\sigma^i = 0 \tag{26}$$

Looking at the first part of the left side:

$$\left[\tfrac{1-\theta^2}{\theta}A^2\left(\sigma^i k_0\right)^{1-\theta}\left(\tfrac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2}-\tfrac{1}{(\tau^i)^2\rho}\right]d\tau^i =$$
$$\left[A\left(\sigma^i k_0\right)^{-\theta}\left(\tfrac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-1}\tfrac{1-\theta}{\theta}k_0\right]d\sigma^i$$

LHS: The first term is only negative if $\theta > 1$ and positive if $\theta < 1$ while the second term is always negative.

RHS: This is only positive if $\theta < 1$ and negative if $\theta > 1$.

So we can conclude that:

- $\frac{d\tau^i}{d\sigma^i} > 0 \qquad if\ \theta > 1$ because:

$$\begin{aligned}(A)\,d\tau^i &= (B)\,d\sigma^i \\ \frac{d\tau^i}{d\sigma^i} &= \frac{B}{A}\end{aligned}$$

And since A and B are negative:

$$\frac{d\tau^i}{d\sigma^i} > 0$$

- $\frac{d\tau^i}{d\sigma^i} = 0 \qquad if\ \theta = 1$ because:

$$\begin{aligned}-\left[\frac{1}{(\tau^i)^2\rho}\right]d\tau^i &= [0]\,d\sigma^i = 0 \\ d\tau^i &= 0 \\ \Rightarrow \frac{d\tau^i}{d\sigma^i} &= 0\end{aligned}$$

Empirically the θ is usually not smaller than one and therefore one can conclude that the individual tax rate is monotonically increasing with the income share. This stays in contrast to the findings of Alesina and Rodrik(1994). The new role of the government overcompensates now the negative effect of capital income taxation. If we assume a θ that is smaller than one, the first term of the lhs in equation (25) will be positive while the second is negative. The rhs of equation (25) will have a negative sign. Therefore the sign for $\frac{d\tau^i}{d\sigma^i}$ can either be positive or negative.

Again using the median-voter theorem as Alesina and Rodrik(1994) the chosen tax rate will be the tax-rate that is preferred by the median voter:

$$-A\left(\sigma^m k_0\right)^{1-\theta}\left(\frac{\rho-(1-\tau^m)(1-\theta)A}{\theta}\right)^{-\theta-1}+\frac{1}{\tau^m\rho}-\frac{A}{\theta\rho}=0$$

If the income is distributed more equally in the economy, the income share of the median voter will be higher and therefore the tax rate is higher but the growth rate will be lower (see equation (23)).

Intuitively, the individuals try to equalize the marginal utility of private and public consumption.

5.3.2 Summary

By using the median voter theorem and assuming that public spending enters the utility function directly, Li and Zou(1998) show that if income is distributed more equally, the preferred tax rate of the median voter is higher meaning that the growth rate will be reduced. As a general result they mentioned that their assumptions as well as the assumptions of Alesina and Rodrik(1994) are not realistic, but the reality should be somewhere in the middle. Therefore, the result of a growth regression should lead to ambiguous results, depending on the focus of the government (consumption or production).

5.4 Concluding Remarks

The "traditional" political economy approach suggests a negative relation between income inequality and economic growth: The models of Alesina and Rodrick(1994) and Persson and Tabellini(1994) are the most cited papers not only because they also found empirical evidence for their theory. Later, Li and Zou(1998) showed, that under different assumptions, the model of Alesina and Rodrick(1994) can lead to exactly the opposite result, leading to a discussion about the validity of the political economy approach.

The model of Li and Zou(1998) is based on the assumption that government spending is used for public consumption only, while the model of Alesina and Rodrik(1994) assumes that government spending is used for production only. Both assumptions are not realistic since in real world it is used for both. So in general the relation between inequality and growth should be ambiguous. Li and Zou(1998) argue that this should lead to insignificant results in growth regressions.

Additionally, one has to mention that this approach is based on a democratic economy, meaning that the models can not be used for countries with an authoritarian regime.

6 The Capital Market Imperfection Models

6.1 Introduction

All possible relations between inequality and economic growth can be shown in a model based on capital market imperfections. While some suggest a negative relation[3] others argue that the relation is ambiguous[4]. An argument for a positive relation between inequality and economic growth is introduced by Foellmi and Oechslin(2008). This subsection will give a brief overview over these models, while the second subsection will introduce the model of Galor and Zeira(1993). Subsection 3 will then summarize the main findings of the capital market imperfection approach..

Aghion and Bolton(1997) came up with a so called “trickle-Down” model that is based on the idea that the capital accumulation of rich people may trickle down to the poor. The mechanism can be found on the capital market. The more capital will be accumulated, the more funds will be available for investment which helps poor individuals to become rich. The interest rate is modeled endogenously by the market of investment funds. The authors show that assuming a fast accumulation of capital, the equilibrium interest rate converges to a fixed level. The interesting question in this case is whether this trickle-down effect that works in favor of equality in wealth distribution is strong enough to cancel out the capital accumulation effect that works in favor of the inequality of wealth distribution. The relation of inequality and economic growth is therefore supposed to be ambiguous in this model.

Galor and Zeira(1993) based their model on imperfect capital markets and the assumption that the interest rate for borrowers is higher than the interest rate for lenders. Therefore the distribution of wealth determines the economic activity. Investment in human capital(indivisible) is therefore only possible for individuals that inherit enough wealth. This leads to an underinvestment in human capital and therefore harms growth not only in the short-run but also in the long-run (due to inter-generational transfers). As a result, the authors mention that a large middle class is important to ensure high economic growth. The model is also robust when determining the wage endogenously. The model will be presented in section 6.

A similar argument is used by Banjeree and Newman(1993). Instead of choosing between working in the skilled or unskilled sector as in the model of Galor and Zeira(1993), individuals have to choose between becoming a worker or an entrepreneur. The authors argue that under the assumption of imperfect credit markets and fixed costs for entrepreneurial activities inequality will lead to an underinvestment in this entrepreneurial activity and

[3] see Galor and Zeira(1993) and Banjeree and Newman(1993)

[4] see Aghion and Bolton(1997)

therefore harms growth.

As seen before, a lot of literature based on the credit market imperfection as the channel through which inequality effects output suggested a negative relation between growth and inequality. The usual argument is that because of credit market imperfections, investment depends on individual wealth and therefore leads to inefficiency because marginal returns do not necessarily equalize. In contrast, Foellmi and Oechslin(2008) came up with a model that shows the adverse result. If inequality is decreased by redistribution from the rich to the middle-class the demand for capital will increase (assuming that the investment function is an increasing but concave function in the initial endowment) and therefore the interest rate goes up. This obviously hurts the poor because they are more dependent on borrowing when they want to invest. Investment of the poor will decrease and since they face the highest marginal returns, this effects growth in a negative way. If this indirect effect is higher than the direct growth-enhancing effect of the redistribution, the overall effect will be negative. This is also in accordance with Galor and Zeira(1993)[5]. This is not only a new way of looking on the relation of inequality and growth, it also questions whether the Gini-coefficient as a measure for inequality in empirical analysis will enter the growth regression positively or negatively. This would also explain the big variety in empirical results of growth regressions as we saw in Part II. The authors suggest for further research to use quantile shares instead of the Gini-coefficient to measure inequality. This allows inequality coming from various parts of the distribution to have different impact on economic growth. This is perfectly in line with the suggestions and empirical findings of Voitchovsky(2005) [6]

6.2 The Model of Galor and Zeira (1993)

The credit market imperfection approach is based on the fact that people might not be able to invest in human capital and therefore loose the opportunity to earn high rates of return. Therefore it might be beneficial for the economy to redistribute from rich to poor since it increases the average productivity of investment and therefore increases growth at least in the short-run. This short run implication is shown in many papers but the long run implication was first introduced in a model of Galor and Zeira(1993)that will be discussed in detail in this subsection.

[5] They show that under the assumption that the production function is not globally concave, the effect of inequality on output is ambiguous.

[6] "The results in this study suggest that growth is facilitated by an income distribution that is compressed in the lower part of the distribution, but not so at the top end. In this view, redistributive policies — such as progressive taxation and social welfare — are likely to facilitate growth through their impact on the bottom of the distribution, and to inhibit growth through their impact on the top of the distribution."(Voitchovsky, 2005)

6.2.1 The Model

The model is an overlapping-generations model with inter-generational altruism. A single good can be produced by two different technologies:

- A technology that uses unskilled labor L^u only. The production function Y^u is therefore given by:

$$Y^u = w^u L^u \tag{27}$$

  where w^u is the marginal productivity in the unskilled sector.

- A technology that uses skilled labor L^s and capital K. The production function Y^s is therefore given by:

$$Y^s = F(K, L^s) \tag{28}$$

  where F is a concave function with constant returns to scale.

Production takes place in each period. For simplicity, investment in capital is made one period in advance, there are no adjustment costs and there is no depreciation of capital. Individuals live for two periods. We assume that they have one parent and one child. Therefore there is no population growth. They can work in the unskilled sector for two periods or they can invest in human capital in the first period and work in the skilled sector in the second period. The amount that is invested in human capital is denoted as h. Each individual is endowed with one unit of labor in each period.

Individuals are altruistic and care about their children. They leave them bequests b that can be used for investment in human capital. Consumption c is assumed to take only place in the second period. The utility of an individual is therefore as follows:

$$U = \alpha \log c + (1 - \alpha) \log b \tag{29}$$

Additionally we assume that all individuals are born with the same abilities and the same preferences. They only differ in their initial endowment that is dependent on the amount inherited.

Two assumptions are made: the rate of interest r is constant over time and the access to capital is free for everyone. Individuals can lend any amount of money on the capital markets at this rate while we assume that borrowers can avoid paying back a loan by moving but this is costly. Lenders can avoid this defaults but this also involves costs. This obviously implies that individuals can not borrow at a rate of r but at a higher rate. The borrowing rate is denoted as $i > r$. Firms can borrow money at r because moving is more complicated for firms. The marginal productivity of capital is therefore in the skilled sector:

$$\frac{dF(K, L^s)}{dK} = r \tag{30}$$

This implies that there is a constant capital-labor ratio that determines the skilled wage w^swhich is constant over time.

The last necessary assumption is that the two labor markets and the good market are perfectly competitive and expectations are rational.

6.2.2 Short-run analysis

- **Capital market equilibrium**

  As we mentioned before the individual borrowing rate i has to be higher than r since the lender has positive costs to trace back the borrower. The basic concept is that a borrower borrows an amount a and pays the interest i_a. In a competitive market this has to cover the interest of the lender as well as the cost of the lender z for tracing back the borrower:

  $$ai_a = ar + z \tag{31}$$

  and lenders choose z high enough to give no incentives of moving to the borrowers:

  $$a(1 + i_a) = \beta z \tag{32}$$

  where $\beta * z$ is the cost of the borrower to evade even if the lender invested an amount z to trace him back. Equation (31) and (32) lead us to the result of the capital market clearing:

  $$i = i_a = \frac{1 + \beta r}{\beta - 1} \tag{33}$$

- **Individual behavior:**

  Looking at the individual investment decision leads us now to 3 cases. We assume that an individual inherits the amount x in the first period of life. Considering now the following cases:

  - An individual that decides not to invest in human capital but inherits an amount x has a Utility as follows:

    $$U^u = \log\left[(x + w^u)(1 + r) + w^u\right] + \varepsilon$$

    where:

    $$\varepsilon = \alpha \log \alpha + (1 - \alpha)\log(1 - \alpha)$$

This individual is obviously a lender, meaning that it leaves a bequest b^u to its descendant:

$$b^u = (1-\alpha)\left[(x+w^u)(1+r)+w^u\right]$$

- An individual that inherits more than it invests in human capital $x > h$ has the following utility function:

  $$U^s = \log\left[(x-h)(1+r)+w^s\right]+\varepsilon$$

  This individual is also a lender, meaning that it leaves a bequest b^u to its descendant:

  $$b^s = (1-\alpha)\left[(x-h)(1+r)+w^s\right]$$

- An individual that inherits less than it invests in human capital $x < h$ has the following utility function:

  $$U^s = \log\left[(x-h)(1+i)+w^s\right]+\varepsilon$$

  This individual is borrower($x-h<0$) and therefore paying the higher interest i. It leaves a bequest b^u to its descendant:

  $$b^s = (1-\alpha)\left[(x-h)(1+i)+w^s\right]$$

It would make sense to assume that not all individuals work in the unskilled sector, therefore we have to assume that the benefit from investing in human capital and then working in the skilled sector is higher than the benefit from working in the unskilled sector:

$$w^s - h(1+r) \geqq w^u + w^u(1+r) = w^u(2+r) \tag{34}$$

With this assumption it is clear, that the potential lenders are going to invest in human capital. For the potential borrowers it depends whether their lifetime utility is higher when they work in the unskilled sector or it is higher in the skilled sector. They would invest in human capital if the following holds:

$$U^s(x) \geqq U^u(x)$$

$$\Rightarrow \qquad x \geq f = \frac{1}{i-r}\left[w^u(2+r)+h(1+i)-w^s\right] \tag{35}$$

Equation (35) (for calculations see also Appendix section B) shows that education is limited to those who at least inherit an amount more than f. Otherwise an individual would prefer to work in the unskilled sector. This implies that only the amount inherited is decisive over whether an individual invests in human capital or not. It also determines the consumption and the bequest. The distribution of inheritances among the individuals therefore determines the amount of unskilled and skilled labor and therefore also the macroeconomic performance of a country. Countries might differ in their initial distribution of wealth and might therefore show different performances. It can be shown that the dynamic process is not ergodic[7] because of the assumption that investment in human capital is indivisible. Therefore the dynamic system does not lead to a single long-run distribution and therefore to different long-run equilibria.

Combining equation (34) and equation (35) we result that $f \leq h$ (for calculation see Appendix section B).

6.2.3 Long-run analysis

As mentioned before, the distribution of wealth in period t, D_t, determines:

- the investment in human capital
- the consumption of the individual
- the bequest of the individual

This basically means it determines the equilibrium in period t. But that is not the whole story. The distribution therefore also determines the distribution of wealth in the next period D_{t+1}:

$$x_{t+1} = \begin{cases} b^u(x_t) = (1-\alpha)\left[(x_t + w^u)(1+r) + w^u\right] & if\ x_t < f \\ b^s(x_t) = (1-\alpha)\left[(x_t - h)(1+i) + w^s\right] & if\ f \leq x_t < h \\ b^s(x_t) = (1-\alpha)\left[(x_t - h)(1+r) + w^s\right] & if\ h \leq x_t \end{cases} \quad (36)$$

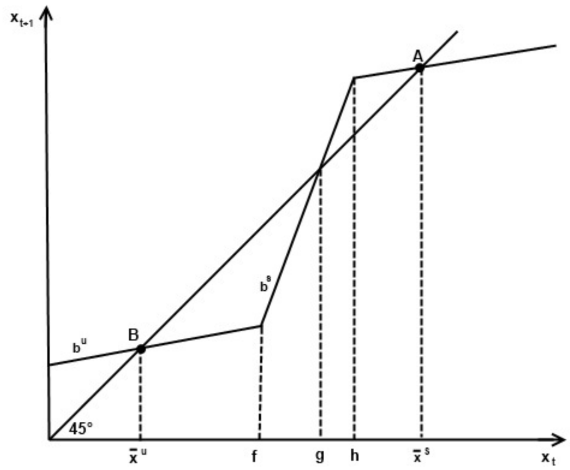
The dynamics in wealth distribution are shown in Figure 6 :

So the interesting question is what happens with individuals that have a typical endowment in period t:

- individuals with an initial endowment $x_t < f$ will work in the unskilled sector and so will do their descendants. The amount inherited converges to a certain level $\bar{x}^u$.

[7] This means that all distributions converge to the same distribution in the long run

Fig. 6: WEALTH DYNAMICS of Galor and Zeira(1993)



Source: Galor, O., & Zeira, J. (1993). Income Distribution and Macroeconomics. Review of Economic Studies 60 , pp. 35-52.

- individuals with an initial endowment $f < x_t < g$ might invest in human capital but in the long-run their descendants end up working in the unskilled sector. The amount inherited converges to a certain level $\bar{x}^u$.

- individuals with an initial endowment $x_t > g$ will invest in human capital and so will do their descendants, generation after generation. The amount inherited converges to a certain level $\bar{x}^s$.

For the determination of g, $\bar{x}^s$ and $\bar{x}^u$ see the Appendix section B.

The long-run dynamics of the model are surprising: it creates a 2 classes society with dynasties that stay in the unskilled sector and dynasties that stay in the skilled sector. Some additional assumptions are necessary to obtain this result. Obviously the slopes of b^u and b^s(when $f \leq x_t < h$) have to be lower than one to get convergence to a stable point. Additionally the slope of b^s has to be higher than one, meaning that the spread between the lending and borrowing interest rate is high enough. Otherwise the convergence would be only to the unskilled class and we end up with a society in which everyone works in the unskilled sector.

Focusing on the initial distribution of wealth D_t, the determination of the size of the two groups in the long-run is not complicated. We know that the number of unskilled workers in the long-run $L^u_\infty$is the same as the number of individuals that inherit less than g in period t.

$$L_\infty^u = L_t(x_t < g) = \int_0^g dD_t(x_t)$$

The long-run level of average wealth W is:

$$W = \frac{L_\infty^s}{L}\bar{x}^s + \frac{L_\infty^u}{L}\bar{x}^u = \frac{L - L_\infty^u}{L}\bar{x}^s + \frac{L_\infty^u}{L}\bar{x}^u = \bar{x}^s - \frac{L_\infty^u}{L}(\bar{x}^s - \bar{x}^u)$$

Since the amount inherited is higher for skilled workers than for unskilled workers ($\bar{x}_s > \bar{x}_u$), the average wealth of the economy is decreasing in the share of unskilled workers $\frac{L_\infty^u}{L}$ but:

$$W = \bar{x}^s - \frac{L_\infty^u}{L}(\bar{x}^s - \bar{x}^u) = \bar{x}^s - \frac{L - L_\infty^s}{L}(\bar{x}^s - \bar{x}^u) = \bar{x}^u + \frac{L_\infty^s}{L}(\bar{x}^s - \bar{x}^u)$$

and is therefore increasing with the share of skilled workers. This has some interesting implications:

- an economy that is initially poor ends up poor
- an economy that is initially rich and the wealth is distributed among many ends up rich
- an economy that is initially rich and the wealth is concentrated in the hands of few ends up poor

The main result states: the bigger the middle class of a country, the better the growth perspectives in the long-run. Economies converge to distinct long-run equilibrium, depending on the initial distribution of wealth.

Whether all these assumptions are reasonable and realistic should be questioned. The authors argue that their result is not dependent on the logarithmic utility function. Also a more general form of the utility function would result in a model where lifetime utility is related monotonically to the bequest and therefore the dynamics and the results do not change.

The assumption of altruistic individuals is more challenging. First, it is not clear whether this reflects reality in a meaningful way and is in the eye of the beholder. Second, there are other ways of modeling the altruistic behavior. This model assumes that utility is a function of consumption and the bequest, while other authors model the altruistic behavior with an utility function that depends on the utility of the next generation. It might be possible that poor individuals save more in order to help their descendants to jump from the unskilled sector to the high skilled sector. Galor and Zeira(1990) show that this might be possible but for very poor individuals it will be not. That means the dynamics will be the same even though the "upper class" might be larger in this case.

The short-run dynamics depend on the assumption of imperfect credit markets but it does not matter how these are modeled. As long as there is a difference in the borrowing rate and the lending rate, these dynamics will be the same since it implies that people with higher wealth have easier access to investment in human capital.

The long-run dynamics are based on the assumption that human capital is indivisible. Galor and Zeira(1993) argue that this assumption is sufficient for the results because Loury(1981) showed that assuming imperfect capital markets and a production function that is smooth and convex in human capital, the distribution of wealth converges to a unique long-run equilibrium. This would imply in the model of Galor and Zeira(1993) that all individuals will invest the same amount in human capital in the long run.

The basic model assumes wages to be constant. This is not a realistic assumption and therefore the model is extended to a model with variable wages. Additionally the variable wage will help to analyze the relation between wealth and equality.

6.2.4 Extended model: Variable wage

To make the model more realistic and to get an insight on the relation between wealth and equality, variable wages are modeled by including a second factor, namely land, to the production function in the unskilled sector:

$$Y^u = G(L^u, N)$$

where N denotes the aggregated amount of land and G is the usual constant returns to scale production function. Assuming that the aggregated amount of land is fixed at a certain level $\overline{N}$, the wage of unskilled workers can be written as follows:

$$w^u = \frac{dG(L^u, \overline{N})}{dL} = H(L^u)$$

where the function H models the diminishing marginal productivity of unskilled labor.

To ease the determination of the supply of unskilled labor the assumption that unskilled individuals work only in the first period of life is useful. We get the labor supply therefore by the number of individuals that won't invest in human capital:

$$S_t = \int_0^{f(w_t^u)} dD_t(x_t)$$

$f(w_t^u)$ is the level of inheritance that changes the individual decision from not investing to investing in human capital as in equation (35) and is determined in the same way:

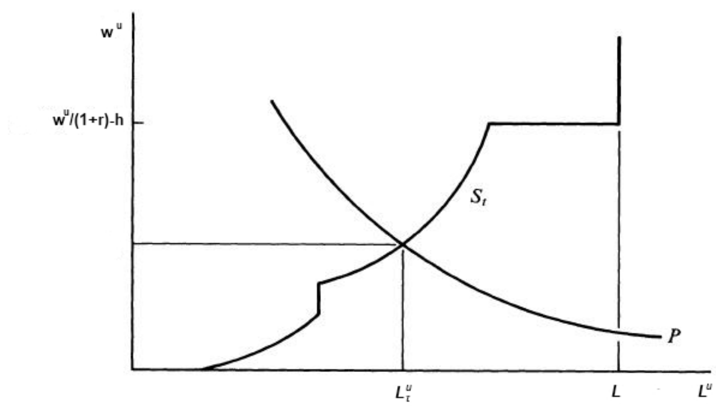
$$f(w_t^u) = \frac{1}{i-r}\left[w_t^u(1+r) + h(1+i) - w^s\right] \tag{37}$$

Looking at equation (37) we see that at a certain level of wage in the unskilled sector individuals are indifferent between investing in human capital or work as unskilled.

If there is a group that inherits the same amount at time t, the supply curve becomes horizontal, if the distribution is such that there are no individuals in a certain area of inheritance (e.g. between $f(w_0)$ and $f(w_1)$), the supply curve becomes vertical between w_0 and w_1. The equilibrium of the unskilled labor market can be seen in figure 7 and determines:

- the amount of unskilled labor,
- the wage of unskilled labor and also
- the number of individuals that invest in human capital

Fig. 7: Unskilled Labor Market with Flexible Wage



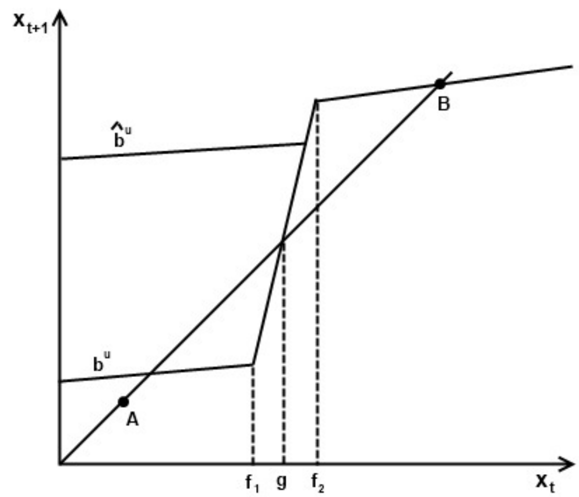
Source: Galor, O., & Zeira, J. (1993). Income Distribution and Macroeconomics. Review of Economic Studies 60 , pp. 35-52.

The only difference in the dynamics compared to the basic model is that wage of unskilled workers is not longer fixed. It is determined endogenously and depends again on the distribution of wealth:

$$x_{t+1} = \begin{cases} b^u(x_t) = (1-\alpha)\left[(x_t + w_t^u)(1+r)\right] & if\ x_t < f(w_t^u) \\ b^s(x_t) = (1-\alpha)\left[(x_t - h)(1+i) + w^s\right] & if\ f(w_t^u) \le x_t < h \\ b^s(x_t) = (1-\alpha)\left[(x_t - h)(1+r) + w^s\right] & if\ h \le x_t \end{cases} \tag{38}$$

The dynamics are shown in figure 8:

Fig. 8: Wealth Dynamics with Flexible Wage



Source: Galor, O., & Zeira, J. (1993). Income Distribution and Macroeconomics. Review of Economic Studies 60 , pp. 35-52.

The only difference to the basic model is now that the b^u line depends on the unskilled wage and therefore shifts if the wage changes. An economy with high equilibrium wage for unskilled labor in period t and $f(w_t^u) > g$ is defined as a developed economy.

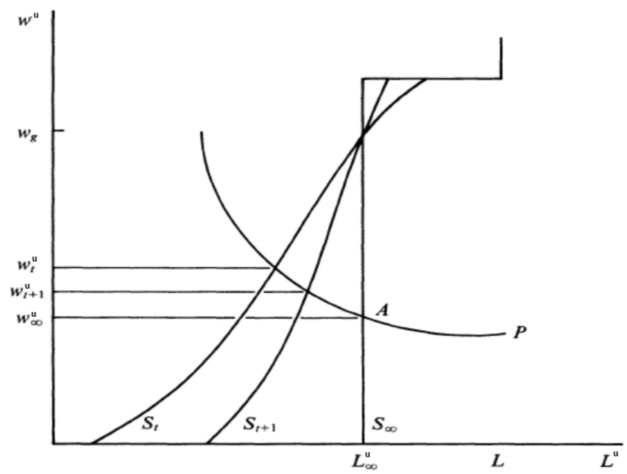
By simple calculation (see Appendix section B) the following holds:

$$w_g \quad = \quad \frac{1}{(1+r)} \frac{[h(1+i) - w^s]\,(i - r - \alpha i + \alpha r - 1)}{(i - \alpha - i\alpha)}$$

An economy is developed if $w_t^u > w_g$. Intuitively this means if the number of people with high inheritances is large. Such a situation is described in figure 8 by the $\hat{b}^u$curve.

A less developed economy is indicated by the b^u line and defined by $w_t^u \leq w_g$ in figure 8. In a less developed economy, individual who inherit more than g leave a bequest that is larger than what they inherit while individuals who inherit less than g leave a bequest lower than what they inherit. Therefore the supply curve gets steeper around w_g in the next period because less people will have a a certain level of inheritance close to $f(w_g)$. This means that the wage shifts downwards as well $(w_t^u > w_{t+1}^u)$ and the bequest gets smaller and smaller. In the long run, the supply curve becomes vertical and the equilibrium point will be point A in figure 8 and 9. The long-run level of wealth for unskilled can be seen in figure 8 at point A. The number of workers in the long-run equals the number of individuals that inherit less than g in period t.

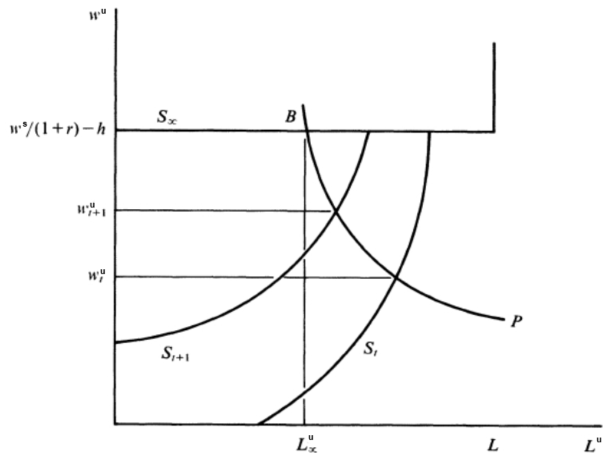
Fig. 9: Dynamics in a Less Developed Economy



Source: Galor, O., & Zeira, J. (1993). Income Distribution and Macroeconomics. Review of Economic Studies 60 , pp. 35-52.

Turning now to a developed economy the dynamics are different. We assume now that $w_t^u > w_g$. Now each individual bequeaths more than it inherits. This means the supply curve shifts to the left. The unskilled wage starts to rise ($w_t^u < w_{t+1}^u$) as shown in figure 10. The long-run supply curve will be horizontal and the equilibrium point will be B. The unskilled wage rate is reached at $\frac{w^s}{(1+r)} - h$ and at this point, $b^n = b^s$. In this equilibrium the net life-time income of all workers (skilled and unskilled) is the same.

Fig. 10: Dynamics in a developed economy



Source: Galor, O., & Zeira, J. (1993). Income Distribution and Macroeconomics. Review of Economic Studies 60 , pp. 35-52.

It is important to note that the long-run equilibrium depends on the distribution of wealth in the starting period. Galor and Zeira(1993) conclude the following from the model:

1. *A less developed country converges to an unequal distribution of income*

2. *A developed country converges toward an equal distribution of income*

6.2.5 Summary

The model of Galor an Zeira(1993) shows basically two important things:

1. Developed countries converge to a more equal distribution of income than less developed countries and the wage differences should be smaller in developed countries.

2. Countries with a more equal initial distribution of wealth tend to grow faster and converge to a higher income level in the long-run

It should be noted that the model might explain the empirical findings in a new way. While e.g. Kuznets(1955) interpreted the differences in the distribution of incomes across countries by the different levels of development, Galor and Zeira(1993) point out that this differences exist because countries converge to different long-run equilibrium.

6.3 Concluding Remarks

As the model of Galor and Zeira(1993) and others show, income and wealth inequality leads to underinvestment in human capital and therefore harms growth under capital market constraints. Depending on the depth of these constraints, the effect will be more or less relevant.

Other CMI models (e.g. Aghion and Bolton(1997)) show that lower inequality has growth enhancing and growth reducing effects, leading to ambiguous results. Additionally, Foellmi and Oechslin(2008) show that redistribution to the poor will be growth enhancing especially in countries with weak capital markets.

7 Other models

There are of course models that are neither based on the political economy approach nor assume imperfect capital markets. Some of them will be presented in this section.

Alesina and Perotti(1996) argue that more inequality tends to increase social discontent and therefore increases the probability of revolutions, mass violence and so on. This means it produces uncertainty in the political environment and threats property rights. This uncertainty of course harms investment and as investment is one of the engines of economic growth, income inequality harms economic growth. The empirical results are in line with the theoretical argument that political instability harms investment. The authors mention that this might explain the relative poor growth performances of South American countries compared to South East Asian countries after World War II. South East Asian countries set in a land reform that led to a decrease in inequality and therefore to a more stable political environment while South American countries failed to set in a land reform and therefore faced more political instability.

García-Penalosa and Turnovsky(2005) developed an AK-model without capital market imperfections. The main argument of their analysis is that wealthier individuals tend to supply less labor than poor ones. Labor supply is endogenous and individuals only differ in their initial capital endowments. The distribution of income and economic growth is therefore determined simultaneously. This implies that all policies that influence the growth rate also have in implication on the distribution of income. Any fiscal policy that increases the labor supply will therefore increase the income inequality(before taxes). The model will be presented in section IV.

In another paper of García-Penalosa and Turnovsky(2006) again assume that labor supply is endogenous and individuals only differ in their initial capital endowments but now using a Ramsey model. Under these assumptions the accumulation of capital tends to reduce wealth inequality but they also show when the elasticity of substitution is high, wealth inequality and income inequality need not necessary move in the same direction towards the steady state. They also show that an increase in productivity has an equalizing effect if the source of inequality is the wealth distribution. A decrease in the population growth rate will decrease income inequality because it leads to a reduction of labor supply and therefore increases the capital-labor ratio. This increases the wage and decreases the return on capital therefore leading to more equality in income.

Halter, Zweimüller and Oechslin(2010) argue that most theories that predict a positive relation between inequality and economic growth are based on economic mechanisms while most of the theories that predict a negative relation are based on political arguments or on human capital. This implies that the positive relation is more valid in the short-run while the negative effect will be only striking in the long-run. This implies that the overall effect in the long-run is ambiguous, while the short-run effect should be positive.

Part IV. Policy Implications

After presenting a wide range of theories on the relation between income inequality and growth the next chapter will present the main policy implications that are pointed out by the theories mentioned in the previous two chapters. Theories on the relation of income inequality and growth are not always in line in their results. Therefore the policy implications differ as well. In the first section the implications of fiscal policy are analyzed while the second section presents the redistributive policy recommendations of the theories mentioned so far.

8 Fiscal Policies

For fiscal policy analysis, a typical AK-model[8] as used by García-Penalosa and Turnovsky(2005) will be presented. A subsidy on investment is added to the model. This subsidy has an growth enhancing effect but has to be financed by a tax (e. g. a capital, income or consumption tax). The model allows to show the effects of such policies on income distribution (not only for income before taxes but also for income after-taxes).

8.1 The model of García-Penalosa and Turnovsky(2005)

8.1.1 The model

Firms The typical production function for each firm j is used for the model:

$$Y_j = A(L^j K)^\alpha (K^j)^{1-\alpha} \qquad 0 < \alpha < 1 \tag{39}$$

where K^j is the firm $j's$ capital stock and L^j firm $j's$ employment of labor. K denotes the average economy-wide capital stock and $L^j K$ is therefore a measure for the efficiency units of labor employed by the firm. Since all firms are supposed to be identically, they choose the same level of labor and capital. Therefore also the economy-wide average capital stock and labor stock are the same as each firms individual capital stock and labor stock ($K^j = K$ and $L^j = L$). Assuming labor market clearing, the average economy-wide working time is $L = 1 - l$ (every individual is endowed with one unit of time that can be used for leisure or for work), where l is the average leisure time.

The aggregated production function can be derived as follows:

[8] The AK-model is an endogenous growth model that states that the production function is linear in the capital stock

$$Y = AL^{\alpha}K \qquad 0 < \alpha < 1 \tag{40}$$

The aggregated production function is linear in the average capital stock. To derive the equilibrium factor prices, we differentiate the aggregated production function with respect to the factors capital and labor:

$$w = \frac{dY}{dL} = \alpha A(1-l)^{\alpha-1}K \tag{41}$$

$$r = \frac{dY}{dK} = (1-\alpha)A(1-l)^{\alpha} \tag{42}$$

While wage is increasing with the aggregated capital stock K and therefore grows with the economy, the return to capital is independent of the capital stock.

Consumers Consumers are initially endowed with a different capital stock K_0^i but do not differ in any other aspect. The individual share in the total capital stock is given as $k^i = \frac{K^i}{K}$ and is distributed with the function $G(k^i)$ with mean $\sum k^i = 1$ and variance σ_k^2 .

A consumer maximizes its life-time utility that depends on consumption and leisure. Assuming an isoelastic utility function leads to the following maximization problem of the consumer:

$$\max \int_0^{\infty} \tfrac{1}{\beta} \left(C_t^i (l^i)^{\eta}\right)^{\beta} e^{-\rho t} dt \tag{43}$$
$$with \ -\infty < \beta < 1 \ , \ \eta > 0 \ and \ 1 > \beta(1+\eta)$$

where the parameter η denotes the elasticity of leisure in utility. The consumers capital accumulation constraint tells that the change in the capital stock over time is the difference between the consumers income and its consumption:

$$(1-s)\frac{dK^i}{dt} = (1-\tau_K)rK^i + (1-\tau_W)(1-l)w - (1-\tau_C)C^i \tag{44}$$

where s denotes a subsidy to investment in capital and τ denotes a tax on capital income (τ_K) , on labor income (τ_W) or on consumption (τ_C). Since the wage rate depends on the aggregated capital stock (see equation (41)), also the individual capital accumulation depends on the aggregated capital stock.

Government Also the government faces a constraint in the model. We assume that the government has to have a balanced budget at each time t. Therefore the following equation has to hold:

$$s\frac{dK}{dt} = \tau_K rK + \tau_W(1-l_i)w + \tau_C C \tag{45}$$

where C denotes the aggregated consumption and l the economy-wide average time of leisure. The intuition behind that equation is that the amount spent for the investment subsidy can not exceed the tax income at any time.

Consumer maximization The maximization problem was already mentioned above. A consumer chooses the rate of consumption, leisure and capital accumulation that maximizes its utility such that the individual capital accumulation equation (44) is fulfilled:

$$\max \int_0^\infty \tfrac{1}{\beta}\left(C_t^i(l^i)^\eta\right)^\beta e^{-\rho t}dt$$
$$s.t.\ (1-s)\tfrac{dK^i}{dt} = (1-\tau_K)rK^i + (1-\tau_W)(1-l_i)w - (1-\tau_C)C^i$$

The solution to this maximization problem is shown in the Appendix in Part D and the result can be stated as follows:

$$\frac{\frac{dC^i}{C^i}}{dt} = \frac{\frac{dC}{C}}{dt} = \frac{\frac{dK^i}{K^i}}{dt} = \frac{\frac{dK}{K}}{dt} = \gamma = \frac{r\left(\frac{1-\tau_K}{1-s}\right)-\rho}{1-\beta} \tag{46}$$

Since all individuals choose the same growth rate for consumption and capital, the growth rate of average capital stock and average consumption will be the same.

Additionally we get the relative labor supply function (for calculations see Appendix Part D) which indeed is responsible for equating the growth rates of individuals:

$$l^i - l = \left(l - \frac{\eta}{1+\eta}\right)(k^i - 1) \tag{47}$$

Since k^i is not changing over time and García-Penalosa and Turnovsky(2005) show that $l > \frac{\eta}{1+\eta}$, individuals that have a higher initial endowment of capital k^i choose more leisure l^i and therefore supply less labor. Intuitively this can be traced back to a lower marginal utility of wealth for wealthier individuals.

Macroeconomic Equilibrium Assuming that the economy is always on its balanced growth path we get the following equilibrium conditions:

- <u>Equilibrium growth rate:</u>

$$\gamma = \frac{r\left(\frac{1-\tau_K}{1-s}\right) - \rho}{1-\beta}$$

- Aggregated consumption-capital ratio: (for calculations see Appendix section D)

$$\frac{C}{K} = \frac{\frac{(1-\tau_W)}{(1+\tau_C)}\alpha A(1-l)^{\alpha-1}l}{\eta}$$

- The goods market equilibrium:

$$\gamma = A(1-l)^{\alpha} - \frac{C}{K}$$

- The government's budget constraint:

$$\begin{aligned} s\frac{dK}{dt} &= \tau_K rK + \tau_W(1-l_i)w + \tau_C C \\ s\gamma &= \tau_K r + \tau_W(1-l_i)\alpha A(1-l)^{\alpha-1} + \tau_C\frac{C}{K} \end{aligned}$$

The equilibrium conditions summarized in two equations:

$$\gamma = \frac{r\left(\frac{1-\tau_K}{1-s}\right) - \rho}{1-\beta} \tag{48}$$

and

$$\begin{aligned} \gamma &= A(1-l)^{\alpha} - \frac{\frac{(1-\tau_W)}{(1+\tau_C)}\alpha A(1-l)^{\alpha-1}l}{\eta} \\ \gamma &= A(1-l)^{\alpha}\left(1 - \frac{\frac{(1-\tau_W)}{(1+\tau_C)}\alpha l(1-l)^{-1}}{\eta}\right) \end{aligned} \tag{49}$$

where equation (49) is derived by combining the goods market equilibrium and the aggregate consumption-capital ratio.

8.1.2 The economy without government intervention:

In this analysis, for simplicity, the government does not intervene in the economic process by taxes or subsidies. The macroeconomic equilibrium conditions will be therefore easier to interpret and will be stated above:

$$\gamma = \frac{r-\rho}{1-\beta} = \frac{(1-\alpha)A(1-l)^{\alpha} - \rho}{1-\beta} \tag{50}$$

and

$$\gamma = A(1-l)^{\alpha}\left(1 - \frac{\alpha l}{\eta(1-l)}\right) \tag{51}$$

Note that the growth rate γ is decreasing in l in both equations:

- Equation (50) intuitively reflects the fact that more leisure reduces output and therefore increases the consumption-output ratio. This decreases the growth rate.
- Equation (51) intuitively reflects the fact that more leisure decreases the productivity of capital. This implies a fall in the return of consumption and therefore leads to a decrease in the growth rate.

8.1.3 Income distribution:

After determining the equilibrium, not only the distribution of capital but also the distribution of income is of interest. The individual income before-taxes is given as the sum of individual capital income and individual wage income ($Y^i = rK^i + w(1-l^i)$) while the average economy-wide income is given as the sum of average economy-wide capital income and average economy-wide wage income ($Y = rK + w(1-l)$). Individual i's relative income y^i is therefore:

$$y^i = \frac{Y^i}{Y} = \frac{rK^i + w(1-l^i)}{rK + w(1-l)}$$

$$\Rightarrow y^i - 1 = \lambda(l)(k^i - 1) \quad where\ \lambda(l) = 1 - \frac{\alpha}{(1+\eta)(1-l)} \tag{52}$$

Calculations are shown in the Appendix in Part D. Equation (52) shows that the distribution of pre-tax income depends on two factors:

- the initial distribution of capital and
- and the chosen amount of leisure in equilibrium (since this determines the factor prices)

García-Penalosa and Turnovsky(2005) show, that as long as the equilibrium is one of positive growth, $0 < \lambda(l) < 1$. Therefore, relative income y^i is strictly increasing in the relative capital endowment k^i. This leads to the conclusion, that even though richer individuals supply less labor, the effect of higher capital endowment is still not offset. But as a consequence, the standard deviation of income across individuals σ^y (as a measure of income inequality) has to be smaller than the standard deviation of capital endowments across individuals σ^K.

As a consequence the following equation holds:

$$\sigma^y = \lambda(l)\sigma^K$$

Given σ^K, σ^y is a decreasing and concave function of the average economy-wide leisure time l because more leisure will increase the wage w but decreases the return on capital r. This lowers the range of total income in the economy. Knowing the equilibrium allocation of labor l and the distribution of initial capital will therefore lead us to the variability of income across individuals.

The same procedure can be done for after-tax income and will lead to almost the same results. The calculations are similar as for the pre-tax income:

$$y_a^i = 1 - \lambda_a(l, \tau_W, \tau_K)(1 - k^i) \tag{53}$$

$$where\ \lambda_a(l, \tau_W, \tau_K) = \lambda(l) + (1 - \lambda(l))(1 - \alpha)\frac{(\tau_W - \tau_K)}{\alpha(1+\tau_W)+(1-\alpha)(1-\tau_K)} \tag{54}$$

and therefore the standard deviation of after-tax income can be written as:

$$\sigma_a^y = \lambda_a(l, \tau_W, \tau_K)\sigma^K$$

Comparing the pre-tax distribution and the after-tax distribution of income, one can see that the latter shows higher dispersion only if the tax on labor is higher than the tax on capital ($\tau_W > \tau_K$).

From equation (53) one can see the effects of the income taxes on the after-tax distribution of income:

- Income taxes influence the consumers labor supply choice, therefore changing the gross factor prices and therefore the before-tax income($\rho(l)$ in equation (54))
- The direct redistributive effects are covered in the other part of equation (54) : A higher tax on capital decreases this part, while a higher tax on labor increases it.

Note that a tax on consumption or an investment subsidy affects the after-tax distribution only indirectly since they only have an impact on the labor supply.

8.1.4 Summarizing the model:

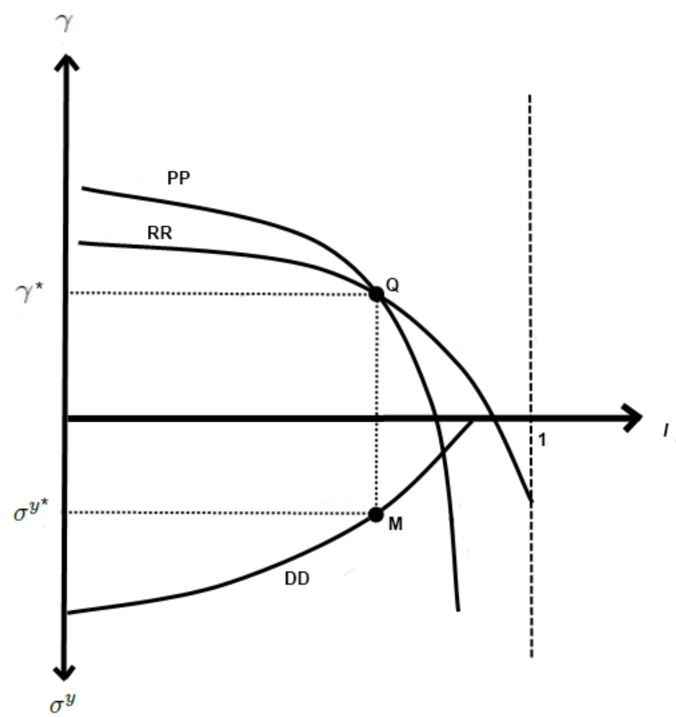
The model can be easily plotted in a graph (see figure 11) , where:

$$RR: \qquad \gamma = \frac{r\left(\frac{1-\tau_K}{1-s}\right) - \rho}{1-\beta}$$

$$PP: \qquad \gamma = A(1-l)^{\alpha}\left(1 - \frac{\frac{(1-\tau_W)}{(1+\tau_C)}\alpha l(1-l)^{-1}}{\eta}\right)$$

$$DD: \qquad \sigma^y = \lambda(l)\sigma^K$$

Fig. 11: Equilibrium Growth, Leisure and Income Distribution



Source: García-Penalosa, C., & Turnovsky, S. (2005). "Growth, Income Inequality and Fiscal Policy: What are the Relevant Tradeoffs?", Working Paper Universités d'Aix-Marseille .

Point Q determines the equilibrium growth rate and the average economy-wide leisure time and point M the standard deviation of pre-tax income.

8.2 Policy implications of the Model

One can summarize the main implications of the model for policy makers in general with three arguments assuming that the initial distribution of capital across individuals is given:

1. *Fiscal policy influences the pre-tax income distribution through its effects on the individual labor supply choice.*

   This can be seen in equation (52) as fiscal policies change the individual labor supply and therefor also the economy wide labor supply.

2. *Any fiscal policy that increases the supply of labor increases pre-tax income inequality and vica versa.*

   Again, equation (52) points out that any policy that increases the labor supply ($(1-l) \uparrow$) increases $\lambda(l)$ and therefore increases y^i, the before-tax income.

3. *A tax on labor and a tax on capital influence the post-tax distribution directly but also indirectly by changing the labor supply. A consumption tax and a investment subsidy only influences the distribution of post-tax income by the latter effect.*

   Equation (53) shows the effects on the after-tax income distribution. We can again see that there is an effect due to a change in labor supply (covered by $\lambda(l)$). Additionally, a tax on labor and a tax on capital directly influence the after-tax income.

The authors mention that the decentralized economy produces a sub-optimal growth rate. An investment subsidy will move the equilibrium closer to the social optimum but there are different ways of financing this subsidy. An investment subsidy will raise the return on capital and therefore favors individuals with high capital endowment. This reverse effect on redistribution might be avoided by the way of financing this subsidy.

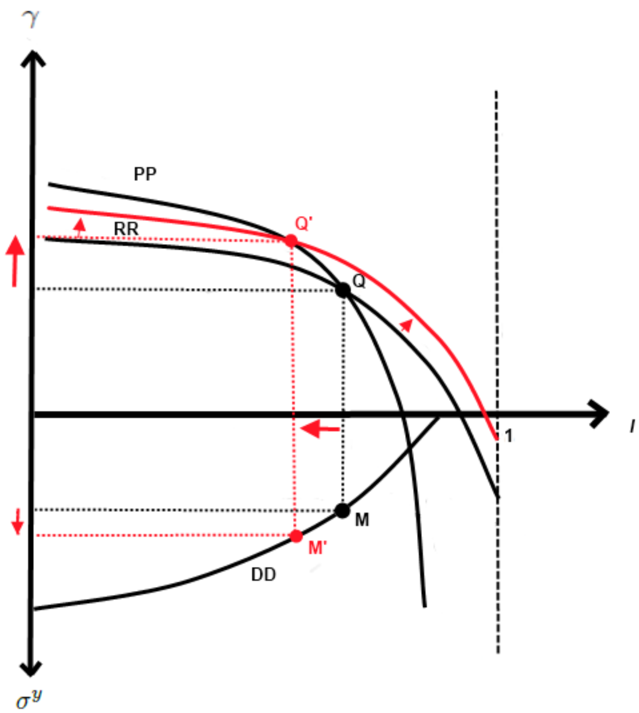
- Financing the investment subsidy by a tax on capital income (see figure 12):

  A higher investment subsidy s will shift the RR-curve upwards ($s \uparrow, 1-s \downarrow, \frac{1}{1-s} \uparrow, \gamma \uparrow$ cet. par.). The equilibrium Q will therefore shift leftwards to the new equilibrium Q' that shows higher growth and less average economy-wide leisure time.

  An increase in the tax rate on capital will have the adverse effect on the RR-curve ($\tau_K \uparrow, 1-\tau_K \downarrow, \gamma \downarrow$). Therefore the overall effect is not clear. It depends on the the size of the capital tax that is needed to finance the investment subsidy. One can show that the effect of the subsidy will dominate the effect of the capital tax. As a result, the overall effects are a higher growth rate, a lower average economy-wide leisure time and additionally M moves to the left till it reaches M', meaning that pre-tax income inequality rises.

  A capital tax obviously ensures that $\lambda_a(l, \tau_W, \tau_K) < \lambda(l)$ meaning that the inequality in after-tax income declines.

Fig. 12: The effects of a capital tax in the García-Penalosa and Turnovsky (2005) model

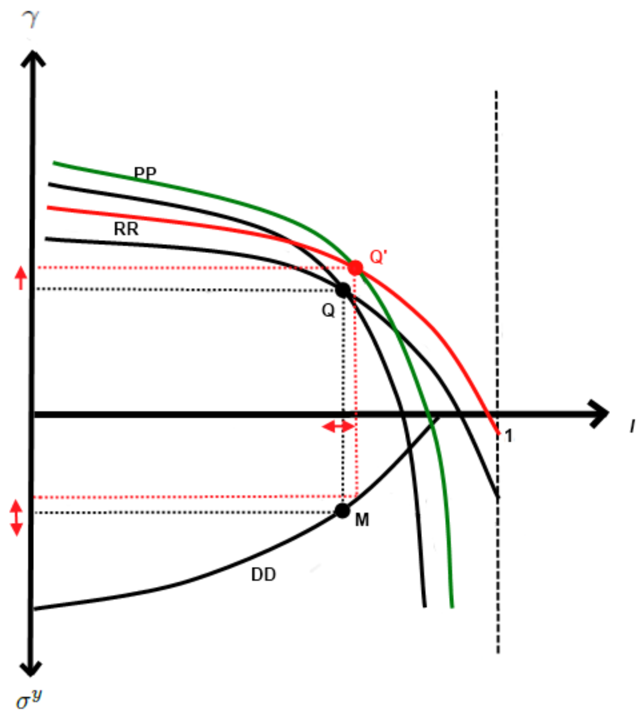


- Financing the investment subsidy by a tax on wage income (see figure 13):

  In this case, the RR-curve shifts upwards because of the investment subsidy (as before). The tax on labor will shift the PP-curve upwards ($\tau_W \uparrow, 1-\tau_W \downarrow, 1-\frac{1-\tau_W}{} \uparrow, \gamma \uparrow$). The overall effect is more complicated in this case. The growth rate will increase while the effect on the average economy-wide labor supply is not clear because the wage tax decreases labor supply while the subsidy that enhances growth will increase the labor supply.

  If the labor supply increases, the pre-tax income inequality will increase and the post-tax income will increase as well. But if the labor supply decreases the pre-tax income inequality will decrease while the post-tax income inequality may increase or decrease.

Fig. 13: The effects of a labor tax in the García-Penalosa and Turnovsky (2005) model



- Financing the investment subsidy by a tax on consumption (see figure 14):

  In this case, $\lambda_a(l, \tau_W, \tau_K) = \lambda(l)$. The RR-curve shifts upwards because of the investment subsidy (as before). The tax on consumption will shift the PP-curve upwards ($\tau_C \uparrow, 1+\tau_C \uparrow, 1-\frac{1+\tau_C}{} \uparrow, \gamma \uparrow$). Again the growth rate will increase. García-Penalosa and Turnovsky(2005) show, that the average economy-wide leisure will decline therefore leading to an increase in pre-tax income inequality. Since the consumption tax has no direct redistributional effect, pre- and post-income inequality are the same and therefore the post-tax income inequality increases as well.

Fig. 14: The effects of a consumption tax in the García-Penalosa and Turnovsky (2005) model

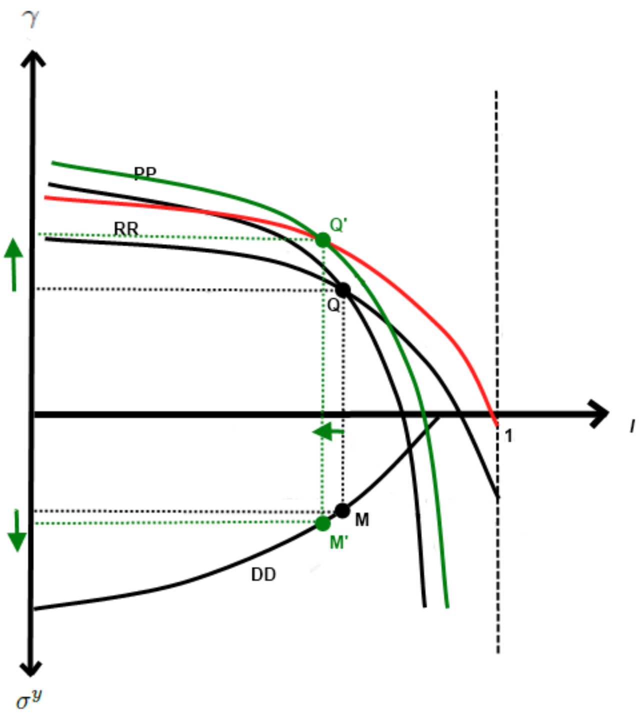


Table 14 summarizes the effects of an investment subsidy that is financed by either a tax on capital, on labor or on consumption.

All three cases lead to a higher growth rate even though the taxes itself would decrease the growth rate. Therefore the growth enhancing effect of the investment subsidy dominates the effect of the taxes. Fiscal policy will increase the pre-tax income in two cases while in the case of a tax on labor the effect is ambiguous. This is in line with the findings of Forbes(2000) and others. The mechanism behind that is that higher growth demands a higher labor supply, therefore reducing the wage rate and increasing the return to capital. This mechanism obviously increases the inequality in income.

As shown, pre-tax and post-tax income inequality need not move in line. The indirect effect (as mentioned above) of the subsidy is always the same, while the direct effect depends on the tax that is chosen to finance the subsidy. The consumption tax has no redistributive effect, the labor tax redistributes towards those with high capital endowment while the tax on capital redistributes to those with lower capital endowment. The question is whether this direct effects can be large enough to offset the positive effect of the investment subsidy on income inequality.

Tab. 14: The effects of an investment subsidy financed by dif. policies

Policy instrument	**growth rate**	**labor supply**	**pre-tax income inequality**	**post-tax income inequality**
wage tax	$\uparrow$	$\uparrow$ *or* $\downarrow$	$\uparrow$ *or* $\downarrow$	$\uparrow$ *or* $\downarrow$
capital tax	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
consumption tax	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

A subsidy financed by a tax on capital is the only policy instrument that enhance growth and might decrease the post-tax inequality. A subsidy financed by a tax on labor stimulates growth and might decrease pre-tax inequality depending on the effect it has on the labor supply. From the pure growth-enhancing point of view, a financing by a consumption tax will be superior to a financing by a tax on labor and in turn to financing by a tax on capital[9].

8.3 Remarks

The model of García-Penalosa and Turnovsky is a model that does not assume capital market imperfections. Labor supply is determined endogenously, individuals differ in their initial capital endowment and therefore growth and income distribution are determined. Therefore policies that influence the growth rate automatically influence the distribution of income. The main mechanism behind that is the wealth effect: Individuals that are endowed with more capital will supply less labor. This means that labor is more equally distributed compared to the capital endowment. Obviously any policy that tries to increase labor supply (and therefore be growth en-

[9] *see: García-Penalosa, C., & Turnovsky, S. (2005). Growth, Income Inequality and Fiscal Policy: What are the Relevant Tradeoffs? Working Paper Universités d'Aix-Marseille*

hancing) raises the relative return to capital and therefore puts more weight on the source of inequality.

There are two limitations of the model:

- The AK-model and the assumption that individuals differ only in the initial endowments of capital leads to the absence of income dynamics. But this is often used for policy analysis (e. g. Alesina and Rodrik(1994)).
- Also other central elements that influence the growth-income inequality relation such as human capital and education are neglected in the model[10].
- The AK-model in general does not predict convergence of per capita GDP levels and there are no transitional dynamics. Every solution is a balanced growth path.

9 Redistributive Policies

Redistributive policy is always an interesting topic. It has deep impact on society and often leads to tough discussions whether redistribution is good or not. Society and policy makers have different views on the topic. This section tries to answer whether redistribution from rich to poor will have a positive or a negative effect on economic growth. From the political point of view, especially the negative relation between income inequality and economic growth has an interesting side result: It would not only decrease income inequality but would also foster growth.

The first subsection will show the distributive policy recommendations of the political economy models, the second subsection will focus on the policy implications of the capital market imperfection models.

9.1 Political-Economy

The policy implications of the political economy models are ambiguous. While Alesina and Rodrik(1994) suggests that equality (and therefore redistribution) is a necessary condition for high growth, Alesina and Perotti(1996) and Li and Zou(1998) argue that the overall effect of redistribution is ambiguous. One should mention, that dynamic analysis is not possible in these models.

Alesina and Rodrik(1994) argue that the government has to provide a public service that is necessary for private production. The public service is financed by a tax on capital and of course every individual would like

[10] for a model with human capital see Galor and Zeira(1993)

to have this public good. The model shows that the higher the income inequality, the higher the chosen tax rate that distorts the economy and the lower economic growth. Redistribution would therefore be growth enhancing for the economy. Li and Zou(1998) show that if this public good is not only used for production but also for consumption services, the effect will be ambiguous leading to the result that the effect of redistribution might foster growth, but it can also be the case that it harms growth. In the theoretical framework of Alesina and Perotti(1996) redistribution in form of higher taxation of capitalists and investors has mainly two effects on the economic performance:

- higher taxation on capitalists and investors will decrease the propensity to invest. As investment is one of the main engines of economic growth the redistributive policy will dampen economic growth.
- the redistribution will lead to more stability in the political system and reduces uncertainty. This indeed should increase investment and therefore foster growth.

The overall effect of fiscal redistribution will foster growth if the indirect positive effect of lower inequality will overweight the direct negative effect on investment. In reality it is not clear that the redistribution will decrease the propensity to invest in general. The overall effect of redistribution on investment can also be in the other direction (see e.g. Foellmi and Oechslin, 2008).

Alesina and Rodrik(1994) show in their model that in an economy with high inequality, the tax rate on capital will be higher than the growth maximizing tax rate. In the model, the capital tax can be seen as a redistributive policy that transfers income to unskilled labor and additionally harms capital accumulation. To enhance growth, it is necessary that policy makers reduce inequality through redistribution. Obviously the government could finance the productive service also with other taxes than a capital tax. The authors point out that the effects of taxes that redistribute from capital to labor will have most likely the same effects as a capital tax.

Li and Zou(1998) show that the model of Alesina and Rodrik(1994) will predict the revers result assuming that the public service will be a consumption service and therefore only enters the utility function of individuals. But the authors point out that the reality lies in between the two models. Public service will be partly used for production services and for consumption services. In this case, the overall effect of inequality on economic growth is ambiguous and therefore also the policy implications are not clear. One might argue that if the government concentrates its activities more on the production side, redistribution will enhance growth while when it concentrates more on the consumption services, redistribution might harm growth.

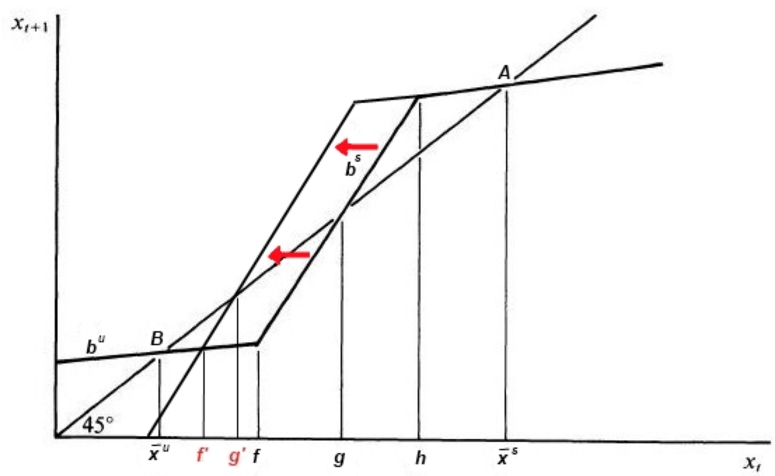
9.2 Capital Market Imperfections

The policy implication that arises from most of the capital-market imperfection models is that there exist redistributive policies that are growth enhancing. Transfers and subsidies for borrowers are an efficient policy tool in this case.

In the Galor and Zeira(1993) model, subsidies for poor and transfers from rich to poor individuals are growth enhancing because more capital-poor people are able to invest in human capital. Obviously this is in line with educational policies such as increased access to education. All this policies will increase growth while they will decrease inequality in income.

Wealth redistribution in the model can increase output and income in the short-run as well as in the long run. But this is not a Pareto improvement. A subsidy on education in the Galor and Zeira(1993) model that is financed by a tax on skilled workers in the next period is shown in figure 15:

Fig. 15: A subsidy on education in the Galor and Zeira(1993) model



This policy will shift the b^s curve leftwards (since it reduces the individual costs for investment in human capital h) and f and g will decrease to $f'$and g'. This policy increases investment and output not only in the short-run but also in the long-run and it will be an Pareto improvement if the collecting costs for debt is higher than the collecting costs for taxes. This seems reasonable when we consider that a tax system already exists.

In the model of Aghion and Bolton(1997), even though there is a trickle-down effect of wealth from the rich to the poor, the distribution of resources in the economy is not efficient. Basically there are two policy implications:

1. The authors show that permanent wealth redistribution from the rich lenders to the middle-class and poor borrowers will lead to more effi-

ciency in the economy. The mechanism behind that is that redistribution equalizes the opportunities to invest in profitable investments. Even though the redistribution from rich to poor will decreases the output and effort of the rich, the increase in output and effort of the poor will be higher. Therefore redistribution leads to more aggregate efficiency in the economy in its steady state.

2. Temporary redistribution can not increase the efficiency of the economy in the steady state, but it can help the economy to achieve the steady state faster.

Foellmi and Oechslin(2008) point out that under the assumption of imperfect capital markets only redistribution to the poorest will increase economic performance. More inequality coming from the bottom-end distribution is therefore less favorable than inequality that arises between upper class and middle class. The argument in favor of this redistributional policies is that the poorest face the highest marginal returns on investment. For redistributive policies, that should work especially against an imperfect capital market, Foellmi and Oechslin(2008) stated: "The model highlights the importance of including not only the middle class but also the least affluent individuals - in particular if the local credit market is not well integrated into the world market"[13]

Part V. Conclusion

The thesis shows that there is a wide range of empirical studies but also numerous theoretical thoughts that do not produce the same results. Therefore the question whether income inequality harms or fosters growth can not be ultimately answered. This contrasting predictions call for further research in this field.

But as an concluding remark, two main thoughts are worthwhile to be mentioned:

1. As Halter, Oechslin and Zweimüller(2010) argue, the contrasting results in theories and empirical results can be traced back to the neglected time dimension. Theories that are based on economic mechanisms and therefore work in the short run usually predict a positive relation between income inequality and economic growth. Theories that are based on social and political mechanisms (e.g. political decisions) which become only effective in the long-run result in a negative relation. The overall effect therefore is ambiguous, depending on which effect is stronger. This might also explain the different empirical results of older and more recent research. Older research was based on short-term effects while the latter focused more on the long-run relation.

2. Galor and Moave(2004) argue that the main engine of growth is in early stages of the development is physical capital accumulation while in later stages it will be human capital accumulation. In this early stages, inequality enhances growth because it distributes resources to people whose marginal propensity to save is higher. In later stages of the economic development as human capital accumulates, there is increasing demand for human capital, which indeed will be then the prime engine of economic growth. Under the assumption of imperfect capital markets more equality will stimulate the investment in human capital and therefore enhance growth. In the last stages of economic development, as individuals get richer and credit constraints become less binding, the effect of the income distribution on the growth process becomes less significant. Thus, for countries with relatively low return to human capital, income inequality might foster growth while in economies with relatively high returns to human capital and where credit constraints are binding, income inequality might be harmful for growth.

Combining these two arguments for policy consideration, the relation between income inequality and economic growth depends not only on a country's stage of economic development but also on the time-dimension the policy targets on.

Policy makers have to keep in mind that there might be a distortion of incentives that goes in line with redistributive policies. Well-targeted and well-financed subsidies and the improvement of the access to education are win-win policies that not only decrease inequality but also enhance growth.

There is still a lot room for improvement in theories and in empirical research. Especially in recent literature, the question arises whether measuring inequality by the Gini coefficient is sufficient. The question of how to redistribute and how much to redistribute is still not answered. But also in the classical literature the general point of view was already the same. As Adam Smith already stated in The Wealth of Nation(1776) : "No society can surely be flourishing and happy, of which the far greater part of the members are poor and miserable."

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Appendix

A. Calculations in the Alesina and Rodrik model (1994):

- The solution of the first maximization problem:

$$\max \qquad \int log\, c^i * e^{-\rho t} dt$$

$$s.t. \qquad \frac{dk^i}{dt} = y^i - c^i = \eta\sigma^i k^i + (r-\tau)k^i - c^i$$

  the variation principle states that a reallocation of consumption between different periods does not improve overall utility. The solution is called the Euler-Equation and has the general form:

$$\frac{\frac{dc^i}{dt}}{c^i} = \frac{(r-\tau)-\rho}{-\frac{U''*c^i}{U'}}$$

  in our case it implies the following:

$$\frac{\frac{dc^i}{dt}}{c^i} = \frac{(r-\tau)-\rho}{-\frac{-(c^i)^{-2}c^i}{(c^i)^{-1}}} = \frac{(r-\tau)-\rho}{\frac{(c^i)^{-1}}{(c^i)^{-1}}} = (r-\tau)-\rho$$

- To obtain the growth maximizing tax rate, maximize the growth rate $\gamma(\tau)$ with respect to the tax rate τ:

$$\max\ \gamma(\tau) = (r-\tau)-\rho = \alpha A \tau^{1-\alpha} - \tau - \rho$$

  - the FOC is as follows:

$$(1-\alpha)\alpha A\tau^{-\alpha} - 1 = 0$$

$$\tau^{\alpha} = (1-\alpha)\alpha A$$

$$\tau^* = [(1-\alpha)\alpha A]^{1/\alpha}$$

- Obtaining the first side condition of the maximization problem 18 by combining equation (14) and (16)

$$\frac{dk^i}{dt} = y^i - c^i = \eta\sigma^i k^i + (r-\tau)k^i - c^i$$

$$\frac{\frac{dk^i}{dt}}{k^i} = (r-\tau)-\rho \qquad \Rightarrow \qquad \frac{dk^i}{dt} = (r-\tau-\rho)k^i$$

– subtracting one in the other will lead us to:

$$(r - \tau - \rho)k^i = \eta\sigma^i k^i + (r - \tau)k^i - c^i$$

$$c^i = (\eta\sigma^i + \rho)k^i$$

- the government maximization problem looks as follows:

$$\begin{aligned} \max \quad & U_i = \int log\, c^i * e^{-\rho t} dt \\ s.t. \quad & \frac{\frac{dk^i}{dt}}{k^i} = \gamma(\tau) = r - \tau - \rho \\ & \frac{\frac{dk}{dt}}{k} = \gamma(\tau) = r - \tau - \rho \\ & c^i = [\eta\sigma^i + \rho]\, k^i \end{aligned}$$

– since we know that also c^i grows at a constant rate we can rewrite our constraints as :

$$c^i = \left[\eta\sigma^i + \rho\right] k^i = c_0^i e^{\gamma t}$$

– plugging this into the Utility function will lead to:

$$\max U_i = \int \left(log\, c_0^i + \gamma t\right) e^{-\rho t} dt$$

– by simple calculations we get:

$$\begin{aligned} U_i &= log\, c_0^i * \int e^{-\rho t} dt + \gamma \int t e^{-\rho t} dt \\ U_i &= log\, \left[\left(\eta\sigma^i + \rho\right) k_0^i\right] \int e^{-\rho t} dt + \gamma \int t e^{-\rho t} dt \\ U_i &= log\, \left[\left(\eta\sigma^i + \rho\right) k_0^i\right] \frac{1}{\rho} + \gamma \frac{1}{\rho^2} \\ U_i &= \frac{1}{\rho}\left[log\, \left(\eta\sigma^i + \rho\right) + log(k_0^i) + \frac{\gamma}{\rho}\right] \end{aligned}$$

– now maximizing this indirect utility function will lead us to:

$$\frac{1}{\rho}\left[\frac{\eta'(\tau)\sigma^i}{(\eta * \sigma^i + \rho)} + \frac{\gamma'(\tau)}{\rho}\right] = 0$$

- Rewriting this will give us the result of the individuals preferred tax rate:

$$\begin{aligned}
\frac{1}{\rho}\left[\frac{\eta'(\tau)\sigma}{(\eta * \sigma^i+\rho)}+\frac{\gamma'(\tau)}{\rho}\right] &= 0\\
-\frac{\gamma'(\tau)}{\rho^2} &= \frac{\eta'(\tau)\sigma^i}{(\eta\sigma^i+\rho)\,\rho}\\
-\frac{\gamma'(\tau)}{\rho} &= \frac{\eta'(\tau)\sigma^i}{(\eta\sigma^i+\rho)}\\
-\gamma'(\tau)\left(\eta\sigma^i+\rho\right) &= \eta'(\tau)\sigma^i\rho
\end{aligned}$$

- we know that $\gamma'(\tau) = \frac{d(r(\tau)-\tau-\rho)}{d\tau} = \frac{d(\alpha A(\tau^i)^{1-\alpha}-\tau-\rho)}{d\tau} = \alpha A(1-\alpha)(\tau^i)^{-\alpha} - 1$ and $\eta'(\tau) = (1-\alpha)^2 A(\tau^i)^{-\alpha}$. Therefore we can write the equation as follows:

$$\begin{aligned}
\left(1-\alpha A(1-\alpha)(\tau^i)^{-\alpha}\right)\left(\eta\sigma^i+\rho\right) &= (1-\alpha)^2 A(\tau^i)^{-\alpha}\sigma^i\rho\\
\frac{\tau^i\left(1-\alpha A(1-\alpha)(\tau^i)^{-\alpha}\right)\left(\eta\sigma^i+\rho\right)}{\tau^i(1-\alpha)^2 A(\tau^i)^{-\alpha}} &= \sigma^i\rho\\
\frac{\left(\tau^i-\alpha A(1-\alpha)(\tau^i)^{1-\alpha}\right)\left(\eta\sigma^i+\rho\right)}{(1-\alpha)^2 A(\tau^i)^{1-\alpha}} &= \sigma^i\rho\\
\frac{\left(\tau^i-\alpha\eta\right)\left(\eta\sigma^i+\rho\right)}{(1-\alpha)\eta} &= \sigma^i\rho\\
\left(\tau^i-\alpha\eta\right) &= \frac{(1-\alpha)\eta\sigma^i\rho}{(\eta\sigma^i+\rho)}\\
\tau^i\left[1-\alpha A(1-\alpha)(\tau^i)^{-\alpha}\right] &= \rho(1-\alpha)\frac{\eta(\tau^i)\sigma^i}{\eta(\tau^i)\sigma^i+\rho}
\end{aligned}$$

B. Calculations in the Galor and Zeira(1993) model:

- Determination of f:

$$\begin{aligned}
U^s &\gtreqqless U^u\\
\log\left[(x-h)(1+i)+w^s\right]+\varepsilon &\gtreqqless \log\left[(x+w^u)(1+r)+w^u\right]+\varepsilon\\
(x-h)(1+i)+w^s &\gtreqqless (x+w^u)(1+r)+w^u\\
x(1+i)-x(1+r) &\gtreqqless w^u(1+r)+w^u+h(1+i)-w^s\\
x(i-r) &\gtreqqless w^u(2+r)+h(1+i)-w^s\\
x &\gtreqqless \frac{\left[w^u(2+r)+h(1+i)-w^s\right]}{i-r} = f
\end{aligned}$$

- Additionally we can show that $f \leq g$ because we know:

$$w^s - h(1+r) \geqq w^u + w^u(1+r) = w^u(2+r)$$

$$\begin{aligned}
\Rightarrow \quad f = & \frac{[w^u(2+r) + h(1+i) - w^s]}{i-r} \\
f \quad \leqq & \quad \frac{1}{i-r}\left[w^s - h(1+r) + h(1+i) - w^s\right] \\
f \quad \leqq & \quad \frac{1}{i-r}\, h\left[-1 - r + 1 + i\right] \\
f \quad \leqq & \quad h
\end{aligned}$$

- Determination of the long-run convergence levels:

$$\begin{aligned}
\bar{x}^u &= (1-\alpha)\left[(\bar{x}^u + w^u)(1+r) + w^u\right] \\
\bar{x}^u - (1-\alpha)\bar{x}^u(1+r) &= (1-\alpha)w^u(1+r) + w^u(1-\alpha) \\
\bar{x}^u\left[1 - (1-\alpha)(1+r)\right] &= w^u(1-\alpha)(1+1+r) \\
\bar{x}^u &= \frac{w^u(1-\alpha)(2+r)}{1-(1-\alpha)(1+r)}
\end{aligned}$$

$$\begin{aligned}
\bar{x}^s &= (1-\alpha)\left[(\bar{x}^s - h)(1+r) + w^s\right] \\
\bar{x}^s - (1-\alpha)\bar{x}^s(1+r) &= (1-\alpha)\left[-h(1+r) + w^s\right] \\
\bar{x}^s\left[1 - (1-\alpha)(1+r)\right] &= (1-\alpha)\left[-h(1+r) + w^s\right] \\
\bar{x}^s &= \frac{(1-\alpha)\left[-h(1+r) + w^s\right]}{\left[1-(1-\alpha)(1+r)\right]}
\end{aligned}$$

$$\begin{aligned}
g &= (1-\alpha)\left[(g - h)(1+i) + w^s\right] \\
g - (1-\alpha)g(1+i) &= (1-\alpha)\left[-h(1+i) + w^s\right] \\
g\left[1 - (1-\alpha)(1+i)\right] &= (1-\alpha)\left[-h(1+i) + -w\right] \\
g &= \frac{(1-\alpha)\left[-h(1+i) + w^s\right]}{\left[1-(1-\alpha)(1+i)\right]} \\
g &= \frac{(1-\alpha)\left[h(1+i) - w^s\right]}{\left[(1-\alpha)(1+i) - 1\right]}
\end{aligned}$$

- An economy with high equilibrium wage for unskilled labor in period t and $f(w_t^u) > g$ is defined as a developed economy:

$$\frac{1}{i-r}\left[w_g(1+r)+h(1+i)-w^s\right]=\frac{(1-\alpha)\left[h(1+i)-w^s\right]}{(1+i)(1-\alpha)-1}$$

$$w_g(1+r)+h(1+i)-w^s=\frac{(1-\alpha)\left[h(1+i)-w^s\right](i-r)}{(1+i)(1-\alpha)-1}$$

$$w_g(1+r)=\frac{(1-\alpha)h(1+i)(i-r)-(1-\alpha)w^s(i-r)}{(1+i)(1-\alpha)-1}-h(1+i)+w^s$$

$$w_g(1+r)=\frac{h(1+i)\left[(1-\alpha)(i-r)-1\right]-w^s\left[(i-r)(1-\alpha)-1\right]}{(1+i)(1-\alpha)-1}$$

$$w_g(1+r)=\frac{\left[h(1+i)-w^s\right]\left[(1-\alpha)(i-r)-1\right]}{(1+i)(1-\alpha)-1}$$

$$w_g=\frac{1}{(1+r)}\frac{\left[h(1+i)-w^s\right](i-r-\alpha i+\alpha r-1)}{(i-\alpha-i\alpha)}$$

C. Calculations in the Li and Zou(1998) model:

- The solution of the first maximization problem:

$$\begin{array}{ll} \max & U^i=\int_0^\infty\left[\frac{\left(c^i\right)^{1-\theta}-1}{1-\theta}+\ln g\right]e^{-\rho t}dt \\ s.t. & \frac{dk^i}{dt}=y^i-c^i=(1-\tau)Ak^i-c^i \end{array}$$

the variation principle states that a reallocation of consumption between different periods does not improve overall utility. The solution is called the Euler-Equation and has the general form:

$$\frac{\frac{dc^i}{dt}}{c^i}=\frac{r-\rho}{-\frac{U''*c^i}{U'}}$$

In our case it implies the following:

$$\frac{\frac{dc^i}{dt}}{c^i}=\frac{r-\rho}{-\frac{\theta(c^i)^{-\theta-1}c^i}{-(c^i)^{-\theta}}}=\frac{r-\rho}{\theta}=\frac{(1-\tau)A-\rho}{\theta}$$

From this equation it follows:

$$k_t^i=k_0^i e^{\frac{(1-\tau)A-\rho}{\theta}t}$$

$$c_t^i = c_0^i e^{\frac{(1-\tau)A-\rho}{\theta}t} = \frac{\rho-(1-\tau)(1-\theta)A}{\theta} k_0^i e^{\frac{(1-\tau)A-\rho}{\theta}t}$$

$$\sigma_0^i = \frac{k_0^i}{k_0}$$

since everything grows at the same level and therefore the income share of an individual is constant over time.

- The maximization problem for a government that maximizes individual i's well-being with respect to τ:

$$\begin{aligned} \max \quad U^i &= \textstyle\int_0^\infty \left[\frac{\left(c^i\right)^{1-\theta}-1}{1-\theta} + \ln g \right] e^{-\rho t} dt \\ s.t. \qquad\qquad k_t^i &= k_0^i e^{\frac{(1-\tau)A-\rho}{\theta}t} \\ c_t^i &= \tfrac{\rho-(1-\tau^i)(1-\theta)A}{\theta} k_0^i * e^{\frac{(1-\tau^i)A-\rho}{\theta}t} \end{aligned}$$

Plugging in the the constraints to the objective function we get:

$$\begin{aligned} U^i &= \textstyle\int_0^\infty \left[\frac{\left(c_0^i e^{\frac{(1-\tau)A-\rho}{\theta}t}\right)^{1-\theta} - 1}{1-\theta} + \ln g \right] e^{-\rho t} dt \\ &= \int_0^\infty \frac{\left(c_0^i e^{\frac{(1-\tau)A-\rho}{\theta}t}\right)^{1-\theta}}{1-\theta} e^{-\rho t} dt - \int_0^\infty \frac{1}{1-\theta} e^{-\rho t} dt + \int_0^\infty \ln g \, e^{-\rho t} dt \end{aligned}$$

– First part:

$$
\begin{aligned}
\int_0^\infty \frac{\left(c_0^i e^{\frac{(1-\tau)A-\rho}{\theta}t}\right)^{1-\theta}}{1-\theta} e^{-\rho t} dt &= \\
\frac{\left(c_0^i\right)^{1-\theta}}{1-\theta} \int_0^\infty e^{\frac{(1-\theta)(1-\tau)A-\rho(1-\theta)-\rho\theta}{\theta}t} dt &= \\
\frac{\left(c_0^i\right)^{1-\theta}}{1-\theta} \frac{\theta}{(1-\theta)(1-\tau)A-\rho(1-\theta)-\rho\theta} \left(e^{\frac{(1-\theta)(1-\tau)A-\rho(1-\theta)-\rho\theta}{\theta}t} \mid_0^\infty\right) &= \\
\frac{\left(c_0^i\right)^{1-\theta}}{1-\theta} \frac{\theta}{(1-\theta)(1-\tau)A-\rho} (0-1) &= \\
\frac{1}{1-\theta} \left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta} k_0^i\right)^{1-\theta} \frac{\theta}{-(1-\theta)(1-\tau)A+\rho} &= \\
\frac{1}{1-\theta} \left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{1-\theta} (k_0^i)^{1-\theta} \left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-1} &= \\
\frac{\left(k_0^i\right)^{1-\theta}}{1-\theta} \left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta} &= \\
\frac{\left(k_0\sigma^i\right)^{1-\theta}}{1-\theta} \left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta} &=
\end{aligned}
$$

– Second part:

$$
\begin{aligned}
\int_0^\infty \frac{1}{1-\theta} e^{-\rho t} dt &= \\
\frac{1}{1-\theta} \int_0^\infty e^{-\rho t} dt &= \\
\frac{1}{1-\theta} \left(\frac{-1}{\rho}\right) \left(e^{-\rho t} \mid_0^\infty\right) &= \\
-\frac{1}{(1-\theta)\rho} (0-1) &= \\
\frac{1}{(1-\theta)\rho}
\end{aligned}
$$

– Third part:

$$
\begin{aligned}
\int_0^\infty \ln g\, e^{-\rho t}dt &= \\
\int_0^\infty \ln\left(\tau A k_t^i\right)\ e^{-\rho t}dt &= \\
\int_0^\infty \ln(\tau)\ e^{-\rho t}dt + \int_0^\infty \ln(A)\ e^{-\rho t}dt + \int_0^\infty \ln\left(k_t^i\right)\ e^{-\rho t}dt &= \\
\ln(\tau)\left(-\tfrac{1}{\rho}\right)\left(e^{-\rho t}\ |_0^\infty\right) + \ln(A)\left(-\tfrac{1}{\rho}\right)\left(e^{-\rho t}\ |_0^\infty\right) + \int_0^\infty \ln\left(k_t^i\right)\ e^{-\rho t}dt &= \\
\ln(\tau)\left(-\tfrac{1}{\rho}\right)(0-1) + \ln(A)\left(-\tfrac{1}{\rho}\right)(0-1) + \int_0^\infty \ln\left(k_t^i\right)\ e^{-\rho t}dt &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \int_0^\infty \ln\left(k_0^i e^{\frac{(1-\tau)A-\rho}{\theta}t}\right)\ e^{-\rho t}dt &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \int_0^\infty \ln\left(k_0^i\right)\ e^{-\rho t}dt + \int_0^\infty \ln\left(e^{\frac{(1-\tau)A-\rho}{\theta}t}\right)\ e^{-\rho t}dt &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \ln\left(k_0^i\right)\ \left(-\tfrac{1}{\rho}\right)\left(e^{-\rho t}\ |_0^\infty\right) + \int_0^\infty \ln\left(e^{\frac{(1-\tau)A-\rho}{\theta}t}\right)\ e^{-\rho t}dt &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} + \int_0^\infty \ln\left(e^{\frac{(1-\tau)A-\rho}{\theta}t}\right)\ e^{-\rho t}dt &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} + \int_0^\infty \tfrac{(1-\tau)A-\rho}{\theta}t\, e^{-\rho t}dt &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} + \tfrac{(1-\tau)A-\rho}{\theta}\int_0^\infty t\, e^{-\rho t}dt &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} + \tfrac{(1-\tau)A-\rho}{\theta}\left[\left(-\tfrac{1}{\rho}e^{-\rho t}t\ |_0^\infty\right) + \int_0^\infty 1\left(-\tfrac{1}{\rho}\right)e^{-\rho t}dt\right] &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} + \tfrac{(1-\tau)A-\rho}{\theta}\left[0 + \left(-\tfrac{1}{\rho}\right)\left(-\tfrac{1}{\rho}\right)\int_0^\infty e^{-\rho t}dt\right] &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} + \tfrac{(1-\tau)A-\rho}{\theta}\left[0 + \left(-\tfrac{1}{\rho}\right)\left(-\tfrac{1}{\rho}\right)\left(e^{-\rho t}\ |_0^\infty\right)\right] &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} + \tfrac{(1-\tau)A-\rho}{\theta}\left[0 + \left(-\tfrac{1}{\rho}\right)\left(-\tfrac{1}{\rho}\right)(0-1)\right] &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} + \tfrac{(1-\tau)A-\rho}{\theta}\left[\left(-\tfrac{1}{\rho^2}\right)\right] &= \\
\tfrac{\ln(\tau)}{\rho} + \tfrac{\ln(A)}{\rho} + \tfrac{\ln\left(k_0^i\right)}{\rho} - \tfrac{(1-\tau)A-\rho}{\theta\rho^2}
\end{aligned}
$$

As a result, we get:

$$
\begin{aligned}
U^i &= \frac{\left(k_0\sigma^i\right)^{1-\theta}}{1-\theta}\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta} - \frac{1}{(1-\theta)\rho} + \frac{\ln\left(\tau^i\right)}{\rho} + \frac{\ln(A)}{\rho} + \\
&\quad \frac{\ln\left(k_0^i\right)}{\rho} - \frac{(1-\tau)A-\rho}{\theta\rho^2} \\
U^i &= \frac{\left(\sigma_i k_0\right)^{1-\theta}}{1-\theta}\left[\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right]^{-\theta} - \frac{A(1-\tau_i)-\rho}{\theta\rho^2} + \frac{\ln\tau^i}{\rho} + const.
\end{aligned}
$$

Now maximizing this utility function leads to the following FOC:

$$-A\left(\sigma^i k_0\right)^{1-\theta}\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-1}+\frac{1}{\tau^i\rho}-\frac{A}{\theta\rho^2}=0$$

- Taking the total differential of this expression:

$$\left(\frac{\partial f}{\partial \tau^i}\right)d\tau^i+\left(\frac{\partial f}{\partial \sigma^i}\right)d\sigma^i=0$$

$$\begin{aligned}
\left(\frac{\partial f}{\partial \tau^i}\right) &= -A\left(\sigma^i k_0\right)^{1-\theta}(-\theta-1)\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2} \\
&\quad \left(\frac{-(-1)(1-\theta)A}{\theta}\right)+(-1)\frac{1}{(\tau^i)^2\rho} \\
&= -A\left(\sigma^i k_0\right)^{1-\theta}(-\theta-1)\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2} \\
&\quad \left(\frac{(1-\theta)A}{\theta}\right)-\frac{1}{(\tau^i)^2\rho} \\
&= -A^2\left(\sigma^i k_0\right)^{1-\theta}(-1)(\theta+1)\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2} \\
&\quad \left(\frac{(1-\theta)}{\theta}\right)-\frac{1}{(\tau^i)^2\rho} \\
&= A^2\left(\sigma^i k_0\right)^{1-\theta}\left(\frac{\theta-\theta^2+1-\theta}{\theta}\right)\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2}- \\
&\quad \frac{1}{(\tau^i)^2\rho} \\
&= A^2\left(\sigma^i k_0\right)^{1-\theta}\left(\frac{1-\theta^2}{\theta}\right)\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2}-\frac{1}{(\tau^i)^2\rho}
\end{aligned}$$

$$\left(\frac{\partial f}{\partial \sigma^i}\right) = -A\left(\sigma^i k_0\right)^{-\theta}(1-\theta)k_0\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2}$$

$$\begin{aligned}
\Rightarrow\ &\left(A^2\left(\sigma^i k_0\right)^{1-\theta}\left(\frac{1-\theta^2}{\theta}\right)\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2}-\frac{1}{(\tau^i)^2\rho}\right)d\tau^t = \\
&\left(A\left(\sigma^i k_0\right)^{-\theta}(1-\theta)k_0\left(\frac{\rho-(1-\tau^i)(1-\theta)A}{\theta}\right)^{-\theta-2}\right)d\sigma^t
\end{aligned}$$

D. Calculations in the García-Penalosa and Turnovsky(2005) model:

- The consumers maximization problem:

$$\max \int_0^\infty \frac{1}{\beta} \left(C_t^i (l^i)^\eta\right)^\rho e^{-\beta * t} dt$$
$$s.t.\ (1-s)\frac{dK^i}{dt} = (1-\tau_K) r K^i + (1-\tau_W)(1-l_i)w - (1-\tau_C)C^i$$

  First we rewrite the problem as follows:

$$\max \int_0^\infty \frac{1}{\beta} \left(C_t^i (l^i)^\eta\right)^\beta e^{-\rho t} dt$$
$$s.t.\ \frac{dK^i}{dt} = \frac{(1-\tau_K) r K^i}{(1-s)} + \frac{(1-\tau_W)(1-l)w}{(1-s)} - \frac{(1-\tau_C)C^i}{(1-s)}$$

  The solution to such a problem is the so called Euler equation and has the form:

$$\frac{\frac{dc^i}{dt}}{C^i} = \frac{\frac{(1-\tau_K)r}{(1-s)} - \rho}{-\frac{U''C^i}{U'}}$$

  The solution therefore looks as follows:

$$\frac{\frac{dc^i}{dt}}{C^i} = \frac{\frac{(1-\tau_K)r}{(1-s)} - \rho}{-\frac{U''C^i}{U'}} = \frac{\frac{(1-\tau_K)r}{(1-s)} - \rho}{-\frac{(\beta-1)(C^i)^{\beta-2}(l^i)^{\eta\beta}e^{-\rho t}C^i}{(C^i)^{\beta-1}(l^i)^{\eta\beta}e^{-\rho t}}} = \frac{\frac{(1-\tau_K)r}{(1-s)} - \rho}{1-\beta}$$

  The three first order conditions are:

  –
$$(C^i)^{\beta-1}(l^i)^{\eta\beta} = -\frac{(1+\tau_C)}{(1-s)}\lambda^i$$

  –
$$\eta (C^i)^\beta (l^i)^{\eta\beta-1} = \frac{(1-\tau_W)K\alpha A(1-l)^{\alpha-1}}{(1-s)}\lambda^i$$

  –
$$r\left(\frac{1-\tau_K}{1-s}\right) = \rho - \frac{\dot{\lambda}^i}{\lambda^i}$$

- Deriving the aggregated consumption-capital ratio from the first two FOCs:

$$(C^i)^{\beta-1}(l^i)^{\eta\beta} = \tfrac{(1+\tau_C)}{(1-s)}\lambda^i \ and$$
$$\eta(C^i)^{\beta}(l^i)^{\eta\beta-1} = \tfrac{(1-\tau_W)K\alpha A(1-l)^{\alpha-1}}{(1-s)}\lambda^i$$
$$(C^i)^{\beta-1}(l^i)^{\eta\beta} = \tfrac{(1+\tau_C)}{(1-s)}\eta(C^i)^{\beta}(l^i)^{\eta\beta-1}\tfrac{(1-s)}{(1-\tau_W)K\alpha A(1-l)^{\alpha-1}}$$
$$(C^i)^{-1}(l^i) = \tfrac{(1+\tau_C)}{(1-\tau_W)\alpha A(1-l)^{\alpha-1}K}\eta$$
$$\tfrac{C^i}{K} = \tfrac{(1-\tau_W)\alpha A(1-l)^{\alpha-1}(l^i)}{(1+\tau_C)\eta}$$
$$\tfrac{C^i}{\frac{K^i}{k^i}} = \tfrac{(1-\tau_W)\alpha A(1-l)^{\alpha-1}(l^i)}{(1+\tau_C)\eta}$$
$$\tfrac{C^i}{K^i} = \tfrac{(1-\tau_W)\alpha A(1-l)^{\alpha-1}(l^i)}{(1+\tau_C)\eta k^i}$$
$$\Rightarrow \tfrac{C}{K} = \tfrac{\frac{(1-\tau_W)}{(1+\tau_C)}\alpha A(1-l)^{\alpha-1}l}{\eta}$$

- we can rewrite the individual capital accumulation as follows:

$$\frac{dK^i}{dt} = \frac{(1-\tau_K)rK^i}{(1-s)} + \frac{(1-\tau_W)(1-l_i)w}{(1-s)} - \frac{(1-\tau_C)C^i}{(1-s)} \tag{55}$$

Dividing both sides by K^i:

$$\begin{aligned}
\frac{\frac{dK^i}{dt}}{K^i} &= \frac{(1-\tau_K)r}{(1-s)} + \frac{(1-\tau_W)(1-l_i)\alpha A(1-l)^{\alpha-1}K}{(1-s)K^i} - \frac{(1-\tau_C)C^i}{(1-s)K^i} \\
\frac{\frac{dK^i}{dt}}{K^i} &= \frac{(1-\tau_K)r}{(1-s)} + \frac{(1-\tau_W)K}{(1-s)K^i}(1-l_i)\alpha A(1-l)^{\alpha-1} - \\
&\quad \frac{(1-\tau_C)}{(1-s)}\left(\frac{(1-\tau_W)\alpha A(1-l)^{\alpha-1}(l^i)}{(1+\tau_C)\eta k^i}\right) &(56) \\
\frac{\frac{dK^i}{dt}}{K^i} &= \frac{(1-\tau_K)r}{(1-s)} + \frac{(1-\tau_W)K}{(1-s)K^i}\alpha A(1-l)^{\alpha-1} &(57) \\
&\quad \left((1-l_i) - \frac{l^i}{\eta}\right)
\end{aligned}$$

The first order conditions imply that $\eta\frac{C^i}{K^i} = \frac{(1-\tau_W)}{(1+\tau_C)}\alpha A(1-l)^{\alpha-1}l^i\frac{K}{K^i}$. Aggregating implies that $K = K^i$ and $l = l^i$and therefore we get

$$\frac{\frac{dK}{dt}}{K} = \frac{(1-\tau_K)r}{(1-s)} + \frac{(1-\tau_W)(1-l_i)\alpha A(1-l)^{\alpha-1}}{(1-s)} - \frac{(1-\tau_C)}{(1-s)} \frac{\frac{(1-\tau_W)}{(1+\tau_C)}\alpha A(1-l)^{\alpha-1}l^i}{\eta} \tag{58}$$

$$\frac{\frac{dK}{dt}}{K} = \frac{(1-\tau_K)r}{(1-s)} + \tag{59}$$

$$\frac{(1-\tau_W)(1-l_i)\alpha A(1-l)^{\alpha-1}}{(1-s)} \left((1-l) - \frac{l}{\eta}\right) \tag{60}$$

Combining (57) and (59) leads us to:

$$\frac{dk^i}{dt} = \frac{d\frac{K^i}{K}}{dt} = \frac{(1-\tau_W)(1-l_i)\alpha A(1-l)^{\alpha-1}}{(1-s)} \left[\left(1-l^i-\frac{l^i}{\eta}\right) - \left(1-l-\frac{l}{\eta}\right)k_t^i\right]$$

One can show that l, l^i are constant over time and the only long-run stability solution is met when $\frac{dk^i}{dt} = 0$[11].
Therefore we get:

$$\frac{dk^i}{dt} = \frac{(1-\tau_W)(1-l_i)\alpha A(1-l)^{\alpha-1}}{(1-s)} \left[\left(1-l^i-\frac{l^i}{\eta}\right) - \left(1-l-\frac{l}{\eta}\right)k_t^i\right] = 0$$

$$\Rightarrow \left(1-l^i-\frac{l^i}{\eta}\right) = \left(1-l-\frac{l}{\eta}\right) * k^i$$

We can rewrite this as:

$$\begin{aligned}
\eta - \eta l^i - l^i &= (\eta - \eta l - l)k^i \\
-(1+\eta)l^i + \eta &= \left(-(1+\eta)l + \eta\right)k^i \\
-l^i + \frac{\eta}{1+\eta} &= \left(-l + \frac{\eta}{1+\eta}\right)k^i \\
-l^i &= \left(-l + \frac{\eta}{1+\eta}\right)k^i - \frac{\eta}{1+\eta} \| * (-1), -l \\
l^i - l &= \left(l - \frac{\eta}{1+\eta}\right)k^i + \frac{\eta}{1+\eta} - l \\
l^i - l &= \left(l - \frac{\eta}{1+\eta}\right)\left(k^i - 1\right)
\end{aligned}$$

[11] see: García-Penalosa, C., & Turnovsky, S. (2005). Growth, Income Inequality and Fiscal Policy: What are the Relevant Tradeoffs? Working Paper Universités d'Aix-Marseille

- Deriving the goods market equilibrium:

$$
\begin{aligned}
\frac{dK}{dt} &= rK + wL - C \\
\gamma &= r + \alpha A(1-l)^{\alpha-1} K \frac{L}{K} - \frac{C}{K} \\
\gamma &= (1-\alpha)A(1-l)^{\alpha} + \alpha A(1-l)^{\alpha} - \frac{C}{K} \\
\gamma &= A(1-l)^{\alpha} - \frac{C}{K}
\end{aligned}
$$

- Deriving the relative individual income y^i:

$$y^i = \frac{Y^i}{Y} = \frac{rK^i + w(1-l^i)}{rK + w(1-l)}$$

We know that:

$$w = \frac{dY}{dL} = \alpha A(1-l)^{\alpha-1} K = \omega K$$

$$r = \frac{dY}{dK} = (1-\alpha) * A * (1-l)^{\alpha} = (1-\alpha) * \Omega$$

and individual labor supply is:

$$l^i = l + \left(l - \frac{\eta}{1+\eta}\right)\left(k^i - 1\right)$$

Plugging in the individual labor supply and the factor prices we get:

$$
\begin{aligned}
y^i &= \frac{Y^i}{Y} = \frac{rK^i + w(1-l^i)}{rK + w(1-l)} = \frac{rK^i + \omega(1-l^i)K}{rK + \omega(1-l)K} \\
y^i &= \frac{rK^i + \omega K(1 - l - \left(l - \frac{\eta}{1+\eta}\right)(k^i - 1))}{rK + \omega K(1-l)} \\
y^i &= \frac{K^i \left(r + \omega \frac{K}{K^i}(1-l) + \omega \frac{K}{K^i}\left(l - \frac{\eta}{1+\eta}\right) * (1-k^i))\right)}{rK + \omega K(1-l)} \\
y^i &= k^i \frac{\left(r + \omega \frac{K}{K^i}(1-l) + \omega \frac{K}{K^i}\left(l - \frac{\eta}{1+\eta}\right)(1-k^i)\right)}{r + \omega(1-l)} \\
y^i &= k^i \frac{\left(r + \omega(1-l) - \omega(1-l) + \frac{\omega}{k^i}(1-l) + \frac{\omega}{k^i}\left(l - \frac{\eta}{1+\eta}\right)(1-k^i)\right)}{r + \omega(1-l)} \\
y^i &= k^i * \left[1 + \frac{\left((1-l)(\frac{\omega}{k^i} - \omega) + \frac{\omega}{k^i}\left(l - \frac{\eta}{1+\eta}\right)(1-k^i)\right)}{r + \omega(1-l)}\right] \\
y^i &= k^i \left[1 + \frac{\left(\omega(1-l)\left(\frac{1-k^i}{k^i}\right) + \frac{\omega}{k^i}\left(l - \frac{\eta}{1+\eta}\right)(1-k^i)\right)}{r + \omega(1-l)}\right] \\
y^i &= k^i \left[1 + \frac{\frac{\omega}{k^i}(1-k^i)\left(1 - l + l - \frac{\eta}{1+\eta}\right)}{r + \omega(1-l)}\right] \\
y^i &= k^i \left[1 + \frac{\frac{\omega}{k^i}(1-k^i)\left(\frac{1}{1+\eta}\right)}{r + \omega(1-l)}\right] \\
y^i &= k^i \left[1 + \frac{\frac{\omega}{k^i}(1-k^i)\left(\frac{1}{1+\eta}\right)}{(1-\alpha)\Omega + \alpha A(1-l)^{\alpha-1}(1-l)}\right] \\
y^i &= k^i + \frac{\omega(1-k^i)\left(\frac{1}{1+\eta}\right)}{(1-\alpha)A(1-l)^{\alpha} + \alpha A(1-l)^{\alpha}} \\
y^i &= k^i + \frac{\omega(1-k^i)}{A(1-l)^{\alpha}(1+\eta)} = k^i + \frac{\alpha A(1-l)^{\alpha-1}(1-k^i)}{A(1-l)^{\alpha}(1+\eta)} \\
y^i &= k^i + \frac{\alpha(1-k^i)}{(1+\eta)(1-l)} \\
\Rightarrow y^i - 1 &= (k^i - 1) + \frac{\alpha(1-k^i)}{(1+\eta)(1-l)} = (k^i - 1)\left(1 - \frac{\alpha}{(1+\eta)(1-l)}\right)
\end{aligned}
$$